A COST TRADE-OFF APPROACH TO PARALLELING OPTIONS IN ASSEMBLY LINE BALANCING

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A COST TRADE-OFF APPROACH TO PARALLELING OPTIONS

IN ASSEMBLY LINE BALANCING

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Dedicated to all the friends I've met at Georgia Institute of Technology, especially to Yurdanur Dülgeroğlu and Ken Hewett.

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TABLE OF CONTENTS

Table of Contents (Continued)

LIST OF TABLES

LIST OF ILLUSTRATIONS

CHAPTER I

INTRODUCTION

Objective, Scope and Limitations

This study attempts a more realistic conceptualization and treatment of the line balancing problem with paralleling than has been previously undertaken. An extensive search of the literature revealed that line balancing with paralleling has been treated by very few authors. Their approaches have been severely limited by simplifications of the problem statement. The thrust of this research was to develop an improved conceptualization of paralleling, including the cost trade-offs involved, and the development of a method of line balancing with paralleling that would incorporate the results of the findings.

Certain limitations were necessary, however, due to the complexity of the problem. One such limitation relates to the way the paralleling options are categorized in Chapter III. Paralleling is defined in a broad sense to be the duplication of any production facility. By duplicating production facilities, the idle time of the workers in the line can be reduced as will be illustrated later in this chapter. The number of paralleling possibilities in a realistic problem, however, can be extremely large. Therefore, a categorization of "lower cost" paralleling options is made, which may exclude some paralleling options worthy of consideration in very special cases.

Another limitation is that the method developed in Chapter V for line balancing with paralleling does not consider possibilities, such as overtime or the sharing of facilities by more than one worker. For a given cycle time, the method provides a balance allowing for paralleling with the objective of minimizing costs, given a maximum allowable number of stations (or maximum allowable idle time). It is useful to have such a balance in order to compare with other balances obtained using conventional methods. Such a balance could also be improved using empirical approaches, as is frequently done with balances obtained using conventional methods.

A critical step toward the objective of this research was the development of methods for cost comparisons between options which involve paralleling. The approach taken is to consider at each iteration of the balancing procedure only those options which "pass" a stated criterion of idle time and have a minimum number of stations involved in the resulting "paralleling complications." If more than one such option results from these initial criteria, different approaches for cost comparison between options are required, one or more of which would be used to make a final selection.

Any method of line balancing with paralleling is likely to be more suited for application to manual flow, unpaced lines because of the fewer complications that may arise as compared with the case of mechanically paced lines. Further, paralleling tends to complicate the material flow through the line with increased buffer stocks becoming a necessity in some cases. It was in the context of manual flow, unpaced lines that this study was performed.

This study is limited to single model, deterministic line balancing. Single model, stochastic line balancing has been approached by several authors, among them Moodie and Young[1] and Freeman[2]. The latter allowed for parallel stations in his method. In the context of this study, with the complexities arising from a broader definition of paralleling, stochastic balancing with paralleling seems unjustified at this stage of development.

Finally, no claim for optimality is made for the balances obtained with the method developed in Chapter V. At any iteration, the list of tasks available for assignment is based upon Kilbridge and Wester's heuristic rule of columns[3]. This method is therefore labelled "heuristic," as it does not search for optimal solutions, but for good balances at low cost.

In summary, the objective of this research was to develop a method for the inclusion of cost criteria in the heuristic solution of line balancing problems with paralleling. In order to reach that objective it was necessary to:

1. Develop a conceptual foundation of paralleling, its potential applications, advantages, and complexities.
2. Develop methods for comparing costs among different options involving paralleling.
3. Develop a heuristic method for line balancing with paralleling incorporating the concepts and methods from steps 1 and 2.

Line Balancing

The line balancing problem may be stated as follows. Given:

1. The manufacturing method.
2. The subdivision of the total work required to be performed

on any unit into tasks with constant times.

3. A set of precedence restrictions for the tasks considered.
4. Other restrictions relative to the specific situation (e.g., zoning, positioning).
5. A desired output per unit time.

determine how the tasks should be assigned to workers and facilities which constitute the line.

The problem has been recognized as complex by many researchers. In 1955 Salveson[4] pioneered attempts to develop a procedure for solving the problem. Many other procedures have been developed since then, most of them with a clear influence of Salveson's formulation. As a result the "classical deterministic line balancing problem" (CDLBP) has been used as a framework for most procedures.

The CDBLP as presented by Freeman[5] is, "Given:

1. A desired cycle time (Inverse of the production rate).
2. Work elements with constant times.
3. Precedence constraints.

Restrictions:

1. Each work element is assigned to a single work station.
2. The precedence constraints are satisfied.
3. The desired cycle time is not exceeded in any station.

Objective: To find an assignment of work elements to work stations that minimizes the idle time".

Assuming one-worker stations, the above is equivalent to:

1. If the number of stations is fixed, minimize the cycle time subject to the restrictions, or

2. If the cycle time is fixed, minimize the number of stations for that cycle time subject to the restrictions.

Salveson's formulation was based on certain simplifying assumptions as he makes clear in his article. One of his assumptions that relates to paralleling[6]:

> If any $a_i > c$, it would be necessary either to use two or more lines or two or more operators for that task. The latter often is unacceptable (at least in this study) because of operator training, supervision and other related difficulties.

Here a_i is the time of task i and c is the cycle time. (The author of this thesis found Salveson's description of the problem very instructive.)

The different types of methods that have been developed to deal with the line balancing problem are:

1. Analytical methods that attempt to find the "optimum balance." Some of the authors that have suggested such methods, are Jackson[7] in 1956,; Held, Karp and Sharesian[8] in 1963; Klein[9] in 1963; and Gutjahr and Nemhauser[10] in 1964.

2. Heuristic methods trading optimality for solution speed and efficiency. Some authors include Kilbridge and Wester[11] in 1961; Helgeson and Birnie[12] in 1961; Arcus[13] in 1963; Hoffman[14] in 1963; Mansoor[15] in 1964; and Moodie and Young[16] in 1965.

3. Empirical methods suggested by some authors aiming to improve a balance obtained previously in a systematic way by manipulating the elements in an intuitive way. Among these

authors are Sawyer[17] in 1970; and Mariotti[18] in 1970.

While maintaining the objective of minimizing the idle time, most of these methods have made contributions by relaxing in some way the restrictive assumptions, or by providing easier or faster methods to obtain a balance.

Line Types

A flow line is a production system in which a large quantity of discrete items are produced by performing a sequence of operations on them as they pass through a series of production facilities.

In principle, it is intended for the material in a flow line to flow regularly with a tendency to minimize the amount of material in progress. The flow pattern is also intended to be efficient with the material moving over minimum distances. The principle of division of labor is also an important characteristic of flow line production.

An assembly line is a flow line in which assembly operations are being performed. However, other operations, such as cutting, drilling, bending, etc., may also be performed in an assembly line.

By model flexibility, assembly lines may be classified as:

1. Single model lines - those devoted to the production of a single model or item.
2. Multi-model lines - those on which two or more similar types of items are produced separately in batches.
3. Mixed model lines - those on which two or more similar items are produced simultaneously.

By pace flexibility, assembly lines may be classified as:

1. Paced lines - those in which the operator's cycle time is constrained, usually by parts being fed to him by a conveyor. The most common type of pacing in assembly lines is pacing with margin[19], where the effect is that the operator is constrained to complete his tasks on average within a certain time period.
2. Unpaced lines - those in which the operator is free of any mechanically-imposed pace or other constraint on the time the worker may allocate to his task(s) on individual items. In an unpaced line an operator takes the necessary time to complete any item before passing it to next station, so that incomplete items should not be produced. The use of buffer stocks of items between stations is an important feature of these lines.

The concepts and methods in this thesis are developed within the context of single model unpaced lines.

It is very important to note that the formulation of the line balancing problem by Salveson was based upon a paced assembly line as a model[20]:

> Each assembly operator must be assigned a number and combination of jobs (parts to assemble) such that the sum of the times required to carry out his assigned tasks is equal to or less than the cycle time. If his assigned work requires an amount greater than the cycle time, he will not be able obviously to perform all of his tasks or he will be unable to maintain his position on the line and will have to fall behind.

Further in the same reference[21]:

> Because the conveyor usually moves at a uniform speed there is ordinarily no opportunity to store the commodities between stations for allowing different cycle times at different stations.

Hence the assembly-line balancing problem is to select a permutation of the tasks into an assembly sequence and a combination of the tasks into "stations" such that a) The selected combinations of tasks satisfy the technological precedence relationships between the tasks, b) The sum of task performance times assigned to every station is equal to or less than cycle time, and c) The sum of the idle times over all stations is minimum.

Eighteen years later Buxey[22] wrote:

....a survey by Lehman[23] revealed that 40 percent of the U.S. companies balanced their lines by trial and error.... and continues one paragraph below[24]:

Lehman's survey probably indicates a correlation between the lack of general application of current techniques and their unsuitability for many unpaced lines. . . .

Paralleling

Paralleling in assembly line balancing involves the assignment of individual tasks to more than one work station, so that each station performs those tasks on only a fraction of the units produced. Many authors equate the term "paralleling" with "paralleling of a station," which involves performing the same tasks in all of two or more "parallel stations." This study is not restricted to that special form of paralleling.

The paralleling concept originates from relaxing the assumption that every task is performed in only one station. One reason for paralleling would be to improve the balance efficiency for a given cycle time by reducing the idle time and therefore reducing the number of work stations required. (One-man work stations will be assumed through the development of this thesis). Another reason for paralleling may be the existence of tasks longer than the desired cycle time. The main disadvantage of paralleling is that it creates complications to the "ideal" of flow line production. Because of paralleling, the flow of material is more complex than a straight line, and the amount of work in progress may increase. Layout difficulties may arise. Workers may be required to perform more different tasks, which is not desirable from the perspective of division of labor. Another disadvantage is that paralleling involves extra costs of investment, operation and maintenance of production facilities.

An example is presented to illustrate the concept of paralleling:

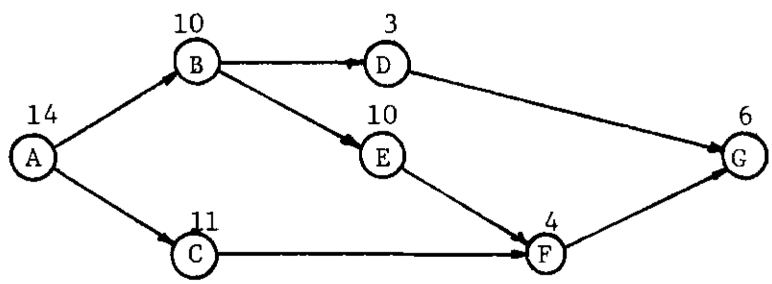


Figure 1. Paralleling, An Example.

If the desired cycle time is 20 and no paralleling is allowed, the minimum-station balances involve four stations. One such solution is:

Station 1:	A	Idle time = 6
Station 2:	B,E	Idle time = 0
Station 3:	D,C,F	Idle time = 2
Station 4:	G	Idle time = 14

If paralleling of task B is allowed a three-station solution can be found:

Station 1: A,B on 1/2 of the units Idle time = 1

Station 2: B on 1/2 of the units, C,D Idle time = 1

Station 3: E,F,G Idle time = 0

And with paralleling of tasks A,B,C and D:

Station 1: A,B,C,D (all on 1/2 of the units) Idle time = 1

Station 2: Same as Station 1 Idle time = 1

Station 3: E,F,G Idle time = 0

The questions remaining are:

1. How can such paralleling solutions be found?
2. Is it worthwhile to parallel in order to reduce the number of stations in the line?

In summary, a redefinition of the assembly line balancing problem is necessary. This study offers a redefinition which includes explicit recognition of paralleling options and cost trade-offs involved.

CHAPTER II

REVIEW OF THE LITERATURE

The first procedure for solving the line balancing problem was developed by Salveson[25] in 1955. Of more transcendency was his definition of the line balancing problem with its restrictive assumptions. His procedure is built on his definition and assumptions.

Many other authors followed with procedures built on Salveson's description of the problem. Salveson's main assumptions were:

1. A known demand,
2. Determined and inalterable technology and production method, and
3. Deterministic work standards.

Salveson defined the assembly line balancing problem as the selection of a permutation of the tasks into an assembly sequence and a combination of the tasks into stations (he assumes one-man stations) such that:

1. The selected combinations of tasks satisfy the technological precedence relationships between the tasks,
2. The sum of the task performance times assigned to every station is equal to or less than the cycle time, and
3. The sum of the idle times over all stations is minimum.

Salveson mentions some refinements that were not incorporated in his procedure:

1. The possibility that task performance times are non-commutative, and
2. The alternative objective of minimizing wages paid instead of idle time.

Salveson utilized a model of mechanically paced assembly lines to define his assembly line balancing problem. It is within this context that he establishes the constraint that each station be assigned a combination of tasks such that the sum of their times is equal to or less than the cycle time.

Kilbridge and Wester[26] in 1961 proposed a manual heuristic method with the advantage of simplicity. The main feature of this technique is the grouping of tasks into "columns" to guide their selection. A "first column number" of a task identifies it with the group of tasks that have the same number of tasks in their longest sequential chain of preceding tasks in the precedence diagram. A "second column number" is assigned to every task. This number identifies the task with the group of elements that have the same number of followers in their longest sequential chain of follower tasks in the precedence diagram. Each task may be transferred into subsequent columns within its range (first column number, last column number). Initially each task occupies the column corresponding to its "first column number." Tasks are selected by their ordering in columns.

Kilbridge and Wester state that judgment and intuition should be used in selecting tasks within columns and in transferring tasks between columns. A person who is balancing the line is expected to make unprogrammed judgmental decisions. This technique imposes the constraint that all task times must be less than or equal to the cycle time.

Arcus[28] in 1963 developed a computerized heuristic method based on the idea of generating a large number of solutions by "biased" random assignment of tasks to stations. The tasks are selected from a fit list consisting of the tasks that are current candidates for assignment to the station under consideration. Arcus worked initially with the simple classical problem. Later he provided for additional constraints and variables:

1. Tasks larger than the cycle time,
2. Tasks which require two workers,
3. Tasks fixed in location,
4. Time to obtain a tool,
5. Time for the worker to change position,
6. Time to change the position of a unit.

His objective was[29]:

> . . . to develop an assembly line balancing method which could be widely used in industry. In a wider sense the objective involved identifying the most important of the measurable variables of machine-paced fixed-unit-interval assembly line systems, and establishing the interrelationship of those variables.

Arcus emphasizes the achievement of minimum idle time in line balancing.

Gutjahr and Nemhauser[27] proposed in 1964 an algorithm based on finding a shortest route in a finite directed network to solve the assembly line balancing problem. The authors report a close relation between their algorithm and other analytical approaches to the line balancing problem which use dynamic programming.

In 1967, Freeman and Jucker[30] discussed additional aspects of the problem. They mentioned the fact, discussed before by other authors, that task times are actually random variables. It was also mentioned that no consideration for interstation inventory or for parallel stations had been given by the line balancing algorithms developed to date. Further, the objective of minimizing idle time is criticized as "inadequate," and the authors propose as a new objective the minimization of total cost per unit produced. They point out that when paralleling of stations is allowed, the cost of duplicating facilities has to be taken into account when evaluating the alternatives.

Freeman[31] in 1967 presents a model in which the required design decisions are:

1. Assignment of elements to stations,
2. Amount of in-process inventory to allow between stations,
3. Whether or not to include parallel stations in the configuration.

He assumes an objective function of the form:

Total cost/unit = labor cost/unit + inventory cost/unit + facility cost/unit + penalty cost/unit

where each term is a function of the expected time between successive items leaving the last station of the line (assumed to be a random variable). The penalty cost is incurred when the system fails to meet the desired output rate. He discusses a solution for his model.

Heskiaoff[32] in 1968 developed a heuristic method to generate a near-optimum balance for telephone set reconditioning lines. The technique is based on the positional weighting technique developed by Helgeson and Birnie[33] and the approach of selecting a minimum cycle time for a specified number of work stations. He considers improving the efficiency of the balance by assigning two operators at a work station[34]:

> If two operators are located at the same work station, each operator can work on every other unit of production that passes the station. The station itself can thus be assigned operations requiring up to twice the normal cycle time.

Heskiaoff's two men-station may be interpreted as equivalent to a station paralleled once.

Mariotti[35] in 1970 showed how better balances may be obtained by improving a systematically developed balance using empirical methods, including element sharing, multiple stations, and multiple lines. Mariotti stresses the fact that by utilizing unprogrammed methods, better solutions may be achieved than with the use of a computer trying to improve a balance. He mentions that an economic evaluation of alternatives should be performed before a specific choice is made.

Buxey, Slack and Wild[36] in 1973 reviewed the state of the art in flow systems design. Referring to heuristic line balancing methods they state[37]:

> Additional constraints can be fed into most heuristic programs if and when needed, although it should be noted that computation may increase and the quality of the solution decrease. However, the most serious drawback with all the methods is that they make no provision for more than one operator at equivalent work stations, which is a common feature of many unpaced lines and is often vital in increasing the production rate up to and beyond the limitation of the longer work elements.

and also[38]:

> Paralleled stations are necessary when element times exceed the nominal cycle time and may on other occasions be desirable in order to achieve satisfactory line balance.

The authors analyze in their article all variables related to line design: buffer stocks, division of labor, parallel stations, allocation of workers to lines, feed interval, tolerance time, and others.

Buxey[39] in 1974 developed two heuristic methods of line balancing with paralleling of stations. One of these methods is based on the positional weight technique developed by Helgeson and Birnie[40]. The other is based on Arcus' random generation method. Buxey begins his article by referring to Lehman's survey[41] in which it was found that approximately 40 percent of U.S. companies balanced their lines by trial and error methods despite the currently available techniques. Buxey attributes this to the inadequacy of these techniques (due to their unrealistic restrictions) when applied to unpaced lines. Buxey discusses the advantages and disadvantages of parallel stations. As advantages he poses:

1. Improvement of balance efficiency.
2. Cycle time can be made smaller than the largest element.
3. Reduced losses due to operator's variability (system loss).

As disadvantages he mentions the material flow and layout difficulties. He also emphasizes that the number of paralleled stations processing the same set of tasks should not be too large, since the cycle time at these stations would become very long, impacting negatively the learning costs. For this, he proposes a limit on the maximum number of tasks at any station. He defines the "maximum multiplicity" of an element as the maximum number of times that the element may be duplicated. This maximum multiplicity must be high enough to make the cycle time possible. As a consequence, the maximum multiplicity of an element restricts the number of times a station containing that element may be paralleled. He explains in his article the overall logic of each method. Some of the most common restrictions in line balancing are contained in his computer programs. He does not, however, consider the cost of duplicating facilities in parallel stations.

In 1975, Pinto, Dannenbring, and Khumawala[42] approached the problem of line balancing with paralleling of tasks allowed for continuous (paced) production lines. The relevant costs of paralleling are assumed to be those of duplication of facilities. They formulate the assembly-line balancing problem with paralleling as selecting the tasks to be paralleled so as to minimize the sum of:

regular time labor cost + duplication of facilities cost + overtime cost required for meeting the desired production rate for the cycle time for which the line is being balanced.

A simplifying assumption is made limiting the number of times a task can be paralleled to one. If, for example, a task is to be paralleled, it would be performed on half of the units in each of the two stations.

Pinto[43] in his dissertation (1975) presents a branch and bound algorithm for the solution of the problem of line balancing with paralleling of tasks mentioned above. This problem is formulated as a mixed-integer program. Based on the same objective function, Pinto also considers another problem: line balancing with paralleling of stations, which he also formulates as a mixed integer program. Two different branch and bound algorithms are developed, one for each of the problems. Therefore, Pinto enlarged the concept of paralleling by applying it to single tasks, as well as to entire work stations. He developed two different methods to deal with paralleling: one for paralleling of tasks and another for paralleling of stations. Pinto also developed a heuristic procedure to reduce computation time while applying his branch and bound methods.

Wild[44] in 1975 suggested a procedure for comparing and selecting between alternatives of mass production systems which could be used in a specific situation. His procedure involves three stages:

1. The selection of a list of possible systems based on considerations of feasibility,
2. The appraisal of such systems in terms of certain quantifiable factors, and
3. The further appraisal of systems on a basis of the non-quantifiable factors.

He concentrates on the analysis of quantifiable factors. Costs and inefficiencies incurred in the different systems are compared. He considers:

1. Learning inefficiency
2. Set up inefficiency
3. Balance loss inefficiency
4. Division of labor loss inefficiency
5. System loss inefficiency
6. Labor recruitment cost
7. Work in progress cost

8. Space Cost
9. Inventory cost
10. Balancing cost
11. Material handling cost
12. Direct labor cost
13. Operation and maintenance cost
14. Capital equipment cost
15. Planning, supervision and control costs
16. Absence cost
17. Quality costs

For each of these costs and inefficiencies, an approach is proposed which would be used to compare between production systems (mechanical lines, non-mechanical lines, collective working, individual assembly). Provision is made for alternative methods of operation of these systems with respect to the manufacture of one or more product types (single model, mixed model and multi-model).

Table 1 summarizes the approaches to paralleling found in the literature.

Table 1. Paralleling in the Literature

Author	Type of Line	Type of Paralleling	Objective	Type of Method	Remarks
Arcus - 1963	Paced	Stations	Min.Idle Time	Heuristic	Paralleling only when tasks larger than cycle time
Freeman-1967	Unpaced	Stations	Min.Costs: Inventory +Facilities +Labor +Penalty	Heuristic	Approach: Stochastic Balancing
Heskiaoff-1968	Paced	Stations	Min.Idle Time	Heuristic	His program assigns up to two operators per station (equivalent to 2 parallel stations)
Mariotti-1970	Paced or Unpaced	Stations and Tasks	Min.Idle Time	Empirical Search	Mentions other costs involved
Buxey-1974	Paced	Stations	Min.Idle Time	Heuristic	Develops two methods
Pinto-1975	Paced	Stations and Tasks	Min.Costs: Facilities+ Normal Labor+ Overtime Labor	Branch and Bound with Heuristics	Two methods: one for paralleling of tasks and one for paralleling of stations

CHAPTER III

PARALLELING OPTIONS

Approach

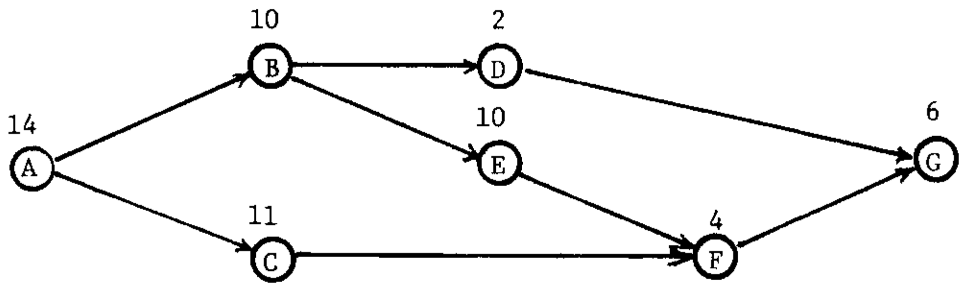
First, a consistent and precise terminology and symbology related to paralleling is developed. Then, some principles of "simple paralleling options" are proposed and illustrated. Finally, some of the limitations of the "simple paralleling options" are discussed.

Terminology and Symbols

The following terms and symbols were used throughout the development of this thesis:

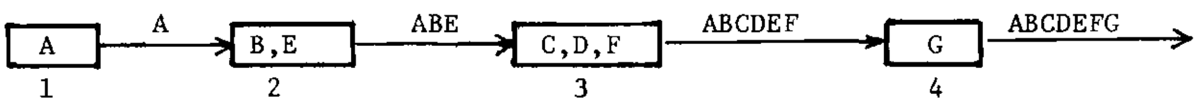
Paralleling of a task. is the performance of that task at more than one station. *Paralleling of a station* involves having two or more stations performing the same tasks on the same fraction of the units. These concepts are illustrated in Figure 2.

Each box represents a work station, and the arrows represent flows between stations. The numbers below the boxes serve only to identify the stations. The letters within the boxes specify which tasks are performed within the station, and the fractions indicate the proportion of units on which each task is performed. For example, in solution III, stations 1 and 2 both perform tasks A, B, C and D, but each station performs them on only half of the units. Station 1 in solution II performs task A on all units and task B on half of the units. The letters on the arrows indicate which task have been performed on units in that flow (or, the *stage of production* of those units).

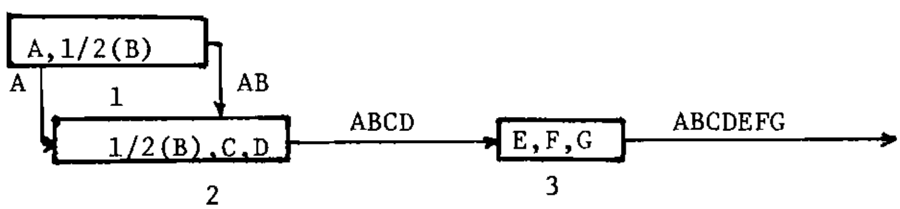


Cycle time = 20

Solution I No paralleling



Solution II Paralleling of task B



Solution III Paralleling of station 1

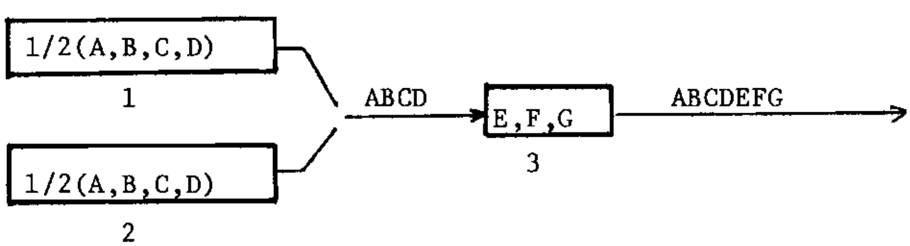


Figure 2. Depicting Paralleling.

A *conventional station* is any station with the following characteristics:

1. Any task performed in that station is performed in no other station (i.e. no paralleled tasks).
2. All units enter the station at the same stage of production, and all units leave the station at the same stage of production.
3. The sum of the tasks assigned to the station is less than or equal to the desired cycle time.

In Figure 2, all stations of solution I are conventional, while only station 3 in both solution II and III is conventional. A non-conventional station is any station lacking one or more of the above characteristics.

Identical stations are two or more stations in which the same tasks are performed on the same fractions of units. In Figure 2, stations 1 and 2 of solution III are identical. It is not necessary, however, that the fraction of units on which each task is performed within each station be equal. A *Unique station* is any station for which there exists no other identical station. In Figure 2, all stations of solutions I and II are unique, while only station 3 of solution III is unique.

An *N-cycle fit* (where N is an integer) is a set of tasks with the following characteristics:

1. The sum of the task times is greater than (N-1) x(desired cycle time) and less than or equal to N x (desired cycle time).

2. Given the current status of the problem solution, it is possible to perform the set of tasks in at least one sequence without violating task precedence relationships. In the precedence diagram of Figure 2, the set $\{A\}$ is a 1-cycle fit, while the sets $\{A,B,C,D\}$ and $\{A,B,C\}$ (among others) are 2-cycle fits; and the set $\{A,B,C,D,E,F,G\}$ is a 3-cycle fit.

An *N-cycle option* is a specific arrangement of an N-cycle fit of tasks within N stations (conventional and/or non-conventional). In Figure 2, solutions II and III are arrangements of the 3-cycle fit $\{A,B,C,D,E,F,G\}$ in 3 stations. They are 3-cycle options.

An *N-cycle paralleling option* is an N-cycle option in which the tasks are arranged within N stations, all of which are non-conventional. Stations 1 and 2 in solutions II and III are specific arrangements of the 2-cycle fit $\{A,B,C,D\}$ in 2 non-conventional stations. They are 2-cycle paralleling options.

The reasons for the definition of N-cycle fit and N-cycle option will become more clear in Chapter V. The line balancing method proposed there searches for fits with a maximum acceptable proportion of idle time. If no acceptable 1-cycle fits are found, the search continues for higher values of N. The acceptable N-cycle fits are later arranged in N-cycle options, which may involve paralleling.

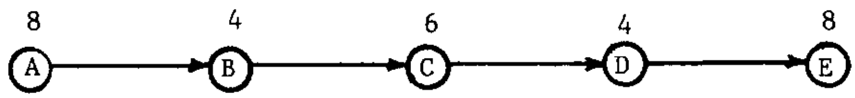
Principles of Simple Paralleling Options

Suppose there is an N-cycle fit from which N-cycle paralleling options are to be arranged. The number of these paralleling options can become unmanageably large. Furthermore, some of the paralleling options may be unreasonably complex, detracting from the benefits of flow line production. With these problems in mind, some principles of "simple paralleling options" are presented here which are intended (a) to limit the search for paralleling options to those that are most promising from an operational and cost viewpoint, and (b) to guide the search for those options.

1. A simple paralleling option will involve a set of non-conventional stations consisting of *either:*
   a. All unique stations, *or*
   b. One or more sets of identical stations, *or*
   c. One or more unique stations and one or more sets of identical stations (this being impossible for 2-cycle paralleling options).

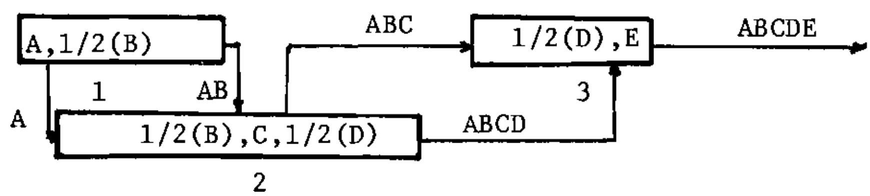
   Figure 3 illustrates the above possibilities for 3-cycle paralleling options arranged from a 3-cycle fit.

2. Given an N-cycle fit generating N-cycle paralleling options, there exists an exhaustive set of "station patterns" which could be considered, each involving N non-conventional unique and/or identical stations. The concept of station patterns is illustrated in Figure 4 for 4-cycle paralleling options (where "U" represents a unique station, "I2" represents one station of a set of two identical stations, "I3" represents one station of a set of three identical stations and "I4" represents one station of a set of four identical stations). Note that the station patterns do not show the

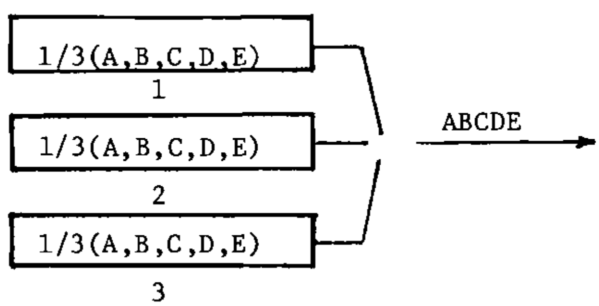


Cycle time = 10

Solution I All unique stations



Solution II One set of 3 identical stations



Solution III One unique station and 1 set of 2 identical stations

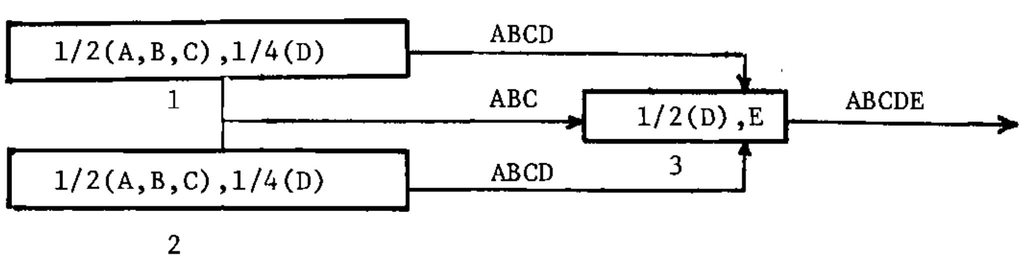


Figure 3. Paralleling Options Arranged from a 3-Cycle Fit.

Pattern 1: Four unique stations.

U U U U

Patterns 2, 3 and 4: Two unique stations and one set of two identical stations.

I2			|		I2		|			I2
	U	U	|	U		U	|	U	U	
I2			|		I2		|			I2

Pattern 5: Two sets of two identical stations.

I2	I2'
I2	I2'

Patterns 6 and 7: One unique station and 1 set of three identical stations.

I3		|		I3
I3	U	|	U	I3
I3		|		I3

Pattern 8: One set of four identical stations.

I4

I4

I4

I4

Figure 4. Station Patterns for 4-Cycle Paralleling Options.

details of the flows, since these may vary from one paralleling option to another within one station pattern (i.e. two or more different paralleling options may conform to the same station pattern). The patterns, however, do imply the general direction of flow through the stations. If we refer to the Figure 4, the material in each pattern would flow consistently from left to right. (Principle 3 will further restrict the flow of materials).

These possible station patterns were also constructed for N-cycle paralleling options with N = 2, 3, ... 7. The number of patterns found for each N is as follows:

N	Number of patterns (p)
2	2
3	4
4	8
5	16
6	32
7	64

The relationship appears to be $p = 2^{N-1}$, though this relationship has not been proven.

3. Once a unit has left a station in a simple paralleling option, it cannot re-enter that station or any station identical to that station.

4. In a simple paralleling option, no more than two types of units (i.e. units in two different stages of production) are allowed to enter any station; also, no more than two types

of units may leave any station. (This principle holds in the paralleling options illustrated in the examples.)

5. For any unique station involved in a simple paralelling option, *either* :

    a. It shares no task with any other station in the paralleling option *or*

    b. It shares only one task with one or more stations (unique or identical) in the paralleling option *or*

    c. It shares only two tasks, one of them with preceeding stations (unique or identical), the other one with following stations (unique or identical).

    Figure 5 illustrates an example of each of these three cases. (Note that in the first example station 2 is non-conventional).

    Also, any station of a set of identical stations in a simple paralleling option relates as a unique station with its non-identicals. That is, *either:*

    a. It shares no task with its non-identicals *or*

    b. It shares one task with one or more of its non-identicals *or*

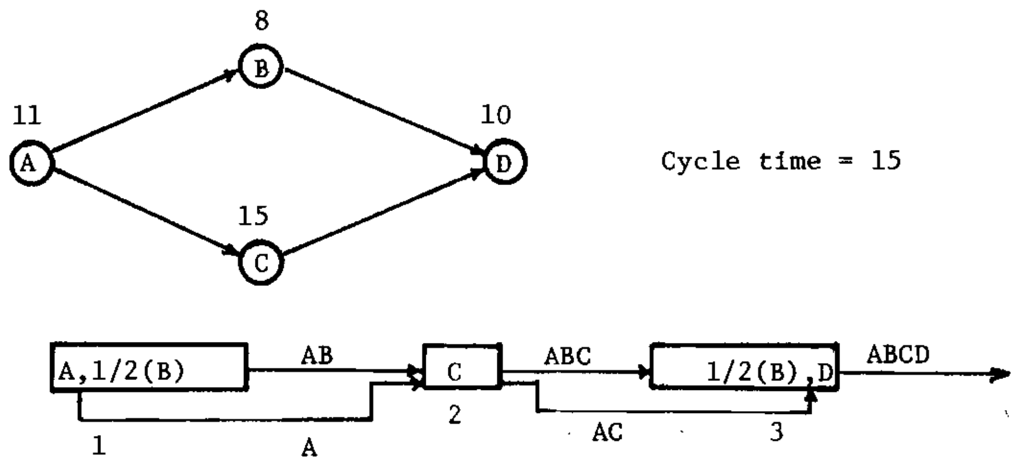
    c. It shares two tasks with its non-identicals, one task with preceeding stations and one task with following stations.

    As a consequence, any pair of stations in a simple paralleling option that are not mutually identical can share at most one task.

These principles would also apply in cases where there are tasks longer than the cycle time. Figure 6 provides an example.

Example:

Unique stations sharing no tasks and one task.



Example:

A unique station sharing two tasks.

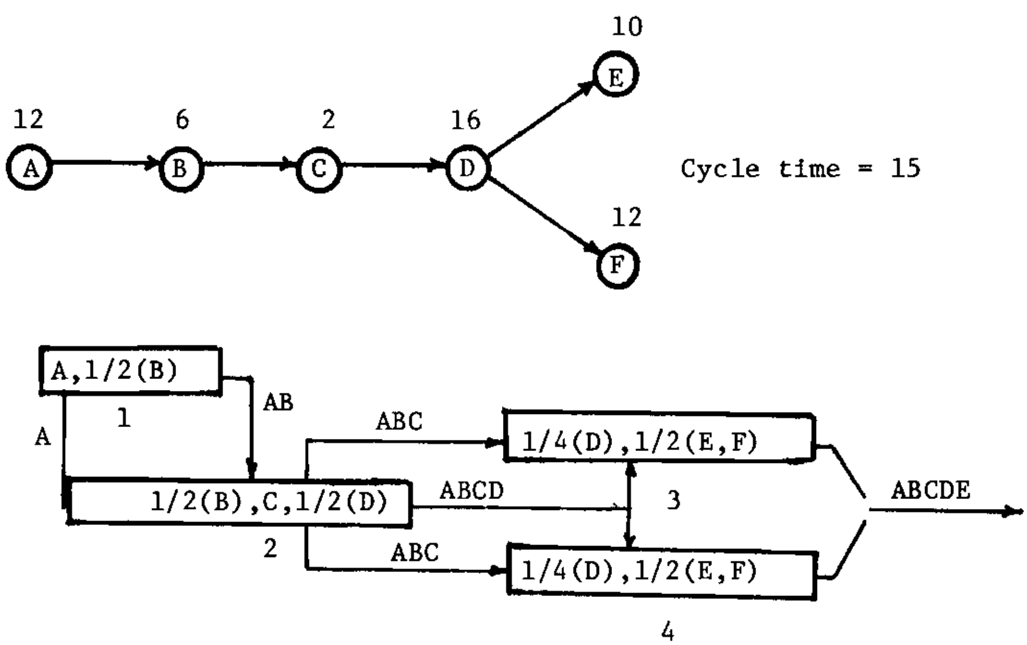
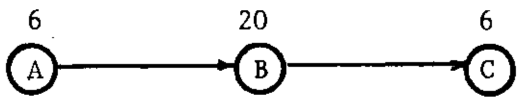


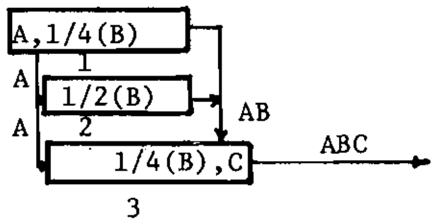
Figure 5. Unique Stations in Simple Paralleling Options.

Example:



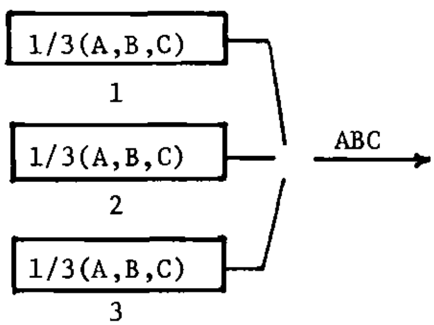
Cycle time = 11

Solution 1 (Simple paralleling option)



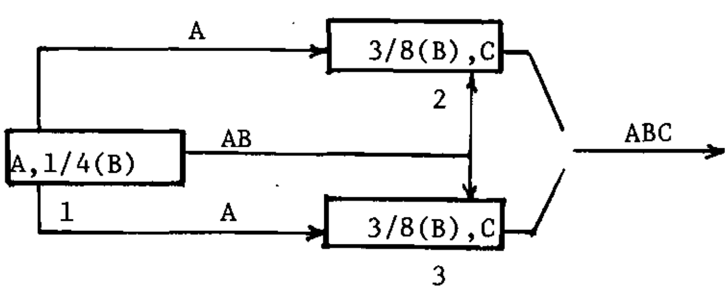
3 unique stations.

Solution 2 (Simple paralleling option)



3 identical stations.

Solution 3 (Simple paralleling option)



1 unique and 2 identical stations.

Figure 6. Principles of Simple Paralleling Options Applied When There are Tasks Longer than the Cycle Time.

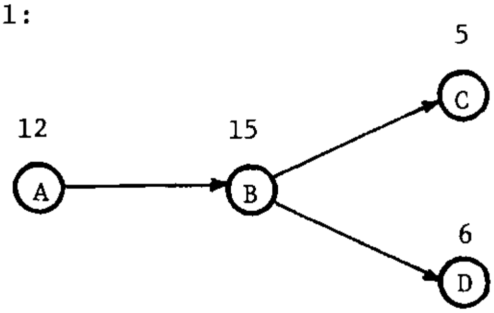
Limitations of the Principles

As mentioned earlier, the principles are intended to limit and guide the search for practical paralleling options. These principles, however, could rule out potentially useful options in some circumstances. Figure 7 shows two representative examples of the limitations of the principles. The first example shows a situation in which "backtracking" could be useful (suppose C and D cannot be performed at the same station). This violates the third principle. The second example shows a situation in which it could be useful to have a pair of unique stations sharing more than one task (suppose paralleling of A is not feasible or a grouping of B and C is desirable). This is in violation of the fifth principle.

Summary

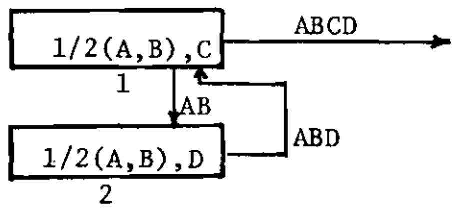
Given a fit which is to be arranged in a paralleling option, the possible ways of doing it may be too numerous and some of them may be too complex. Some principles have been proposed to limit and guide the search for practical alternatives. The principles as stated may rule out useful paralleling options in very specific cases.

Example 1:

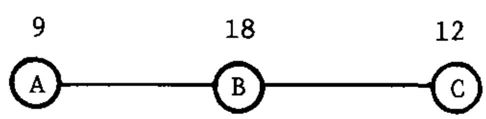


Cycle time = 20

Not a simple paralleling option.



Example 2:



Cycle time = 20

Not a simple paralleling option.

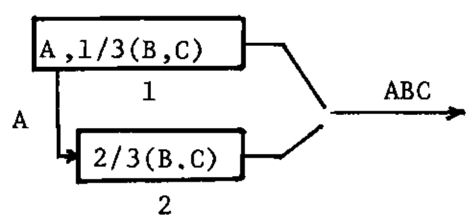


Figure 7. Two Non-Simple Paralleling Options.

CHAPTER IV

COST COMPARISON METHODS

Approach

Initially, the types of costs that are most likely to be affected by paralleling are discussed. Then, based on cost criteria, methods to compare between N-cycle paralleling options arranged from one N-cycle fit are developed. It is then shown how those same methods would be used to compare between N-cycle options in general arranged from the same N-cycle fit.

Costs of Paralleling

One reason for paralleling is that it may improve the efficiency of a balance for a given cycle time by reducing the number of stations required. Another reason may be the existence of tasks longer than the cycle time. However, paralleling involves duplication of facilities and also complicates the flow line, with the subsequent impact on the costs of the line.

The costs that are most likely to be increased by paralleling are as follows:

1. Production equipment. The initial cost of the duplicated production equipment. "Equipment," taken here in a broad sense, includes tools and fixtures. It also includes materials handling equipment used to store or move material *within* a station.

2. Material Handling Equipment. Paralleling generally involves more complex material flows than would otherwise be required. This may result in increased cost to purchase additional materials handling equipment to move material *between* stations.
3. Buffer stocks . The more complex material flows will generally require increased buffer stocks (with the possible exception of parallel stations with units at one stage of production entering the station and units at one stage of production leaving the stations[45]). Consider for example the difficulty of operating Solution I in Figure 3 without buffer stocks of units in each stage of production between the stations.
4. Floor space. Additional floor space may be required for the extra production and materials handling equipment and for the buffer stocks. This involves an initial cost or continued cost (if space is rented) for extra space.
5. Wage rates. Paralleling tends to increase the number of different tasks an operator must perform. Also, paralleling of a high skill task may require a higher wage at the stations performing that task than if that task had not been paralleled.
6. Training. The costs of training new workers is likely to be higher because of the larger number of different tasks per station.
7. Supervision. The complexities of paralleling are likely to involve more supervision of operatives. Also, quality control costs may be higher.

8. Indirect costs, including utilities, insurance, taxes, indirect labor and maintenance and repair costs due to extra production and materials handling equipment, floor space and buffer stocks.
9. Other costs, such as:
   - Quality costs
   - Learning costs
   - Set up costs

Paralleling and Combining Tasks at Stations

The influences that the different ways of combining tasks at stations may have on the costs of the line have been recognized by several authors, including Salveson who pioneered the development of procedures to solve the line balancing problem. Of these costs, the ones that are assumed here to be most relevant are:

1. Investment in production and material handling equipment
2. Wages
3. Training

Other costs such as buffer stocks and supervision, are not likely to vary as much due to this "interaction" among tasks performed at a station.

The usual way to deal with those interactions has been to set initial restrictions in the problem assuming certain high cost combinations as infeasible and also to set initially predetermined groupings of tasks (i.e. task A and B are to be assigned to the same station) when such groupings are accepted to be highly desirable. In this study it is assumed that those highly favorable or unfavorable combinations are set at the beginning as groupings and restrictions, although the effect of the other "less obvious" interactions will also be taken into account.

Cost Comparisons Between N-Cycle Paralleling Options Arranged from One N-Cycle Fit

Given an N-cycle fit which is to be arranged in N non-conventional stations, several simple N-cycle paralleling options may be feasible. This is because:

1. There exist 2^{N-1} different patterns of non-conventional stations which could be tried for arranging the tasks.
2. Also, for most of those patterns, different arrangements of tasks could be tried for each pattern depending on the flexibility of the precedence diagram and other restrictions on the allocation of tasks to stations. (When the pattern is one set of identical stations only one arrangement is possible).

Given a set of feasible N-cycle paralleling options from one N-cycle fit several steps may be taken to determine which is the most economical.

Initial Screening

Initially, a screening among the paralleling options could be useful to eliminate at least some of them from further consideration. The screening involves ordinal comparisons between paralleling options. The approach is to eliminate the options that are clearly inferior to others in terms of costs. This type of comparison is effective provided the relative importance of each of the relevant costs can be assessed for the specific line being designed. An example is presented in Figure 8. In the example, the designer is considering 3-cycle paralleling options. Let us suppose that he is considering the five paralleling options shown in the figure.

In the example, note that three identical stations are not feasible because of the restriction that A and B can be performed at only one station.

The most efficient and easy way to make the ordinal comparisons is to *compare first within subsets of the paralleling options (P.O.) which follow the same pattern.* In this example there are two of such subsets: (a) P.O. 1, 2 and 5, which follow a pattern of three unique stations, and (b) P.O. 3 and 4, which follow a pattern of one unique and two identical stations. To show how this could be done, suppose the designer clearly prefers either P.O. 1 or 2 to P.O. 5 because of material handling considerations, higher wages of P.O. 5 and higher training costs of P.O. 5. He does not consider other differences as relevant, and he proceeds to eliminate P.O. 5. Further, suppose that the designer clearly prefers P.O. 1 to P.O. 2 based on considerations such as: (a) The skills required to perform task F matches better those skills required to perform A and B as opposed to the skills required to perform C with A and B, (b) There would be some savings when duplicating F rather than C in the first station, and (c) The fraction 1/2 is easier to deal with than the fraction 3/5 in controlling the line. The designer then proceeds to eliminate P.O. 2 from further consideration. Suppose also that when comparing within subset II, he prefers P.O. 3

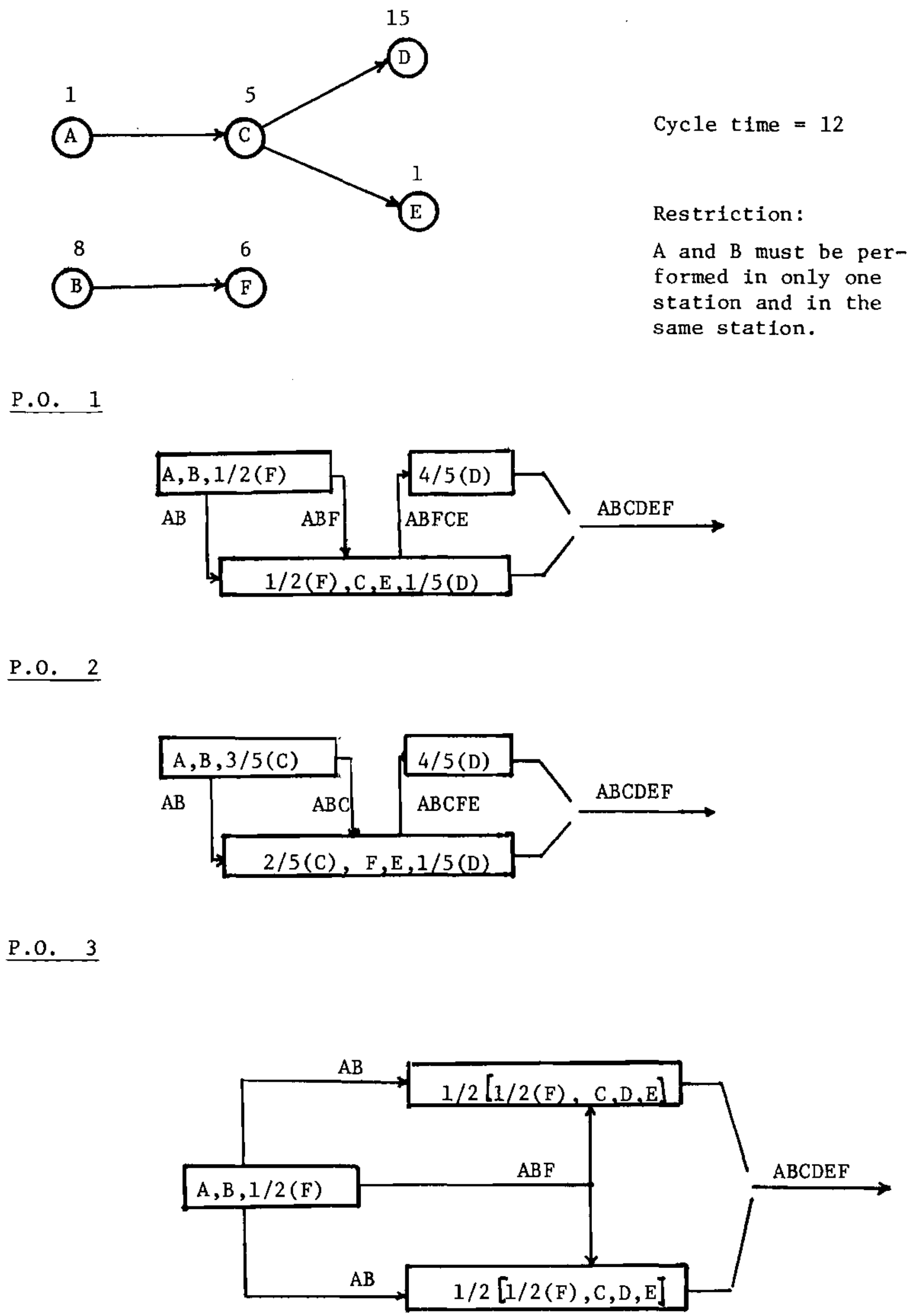
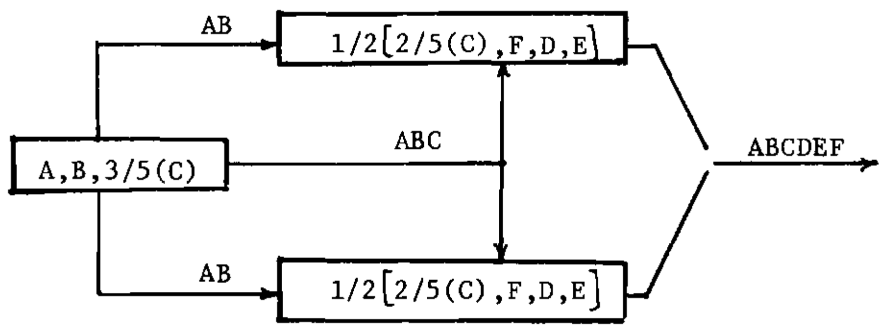


Figure 8. Screening Paralleling Options. An Example.

P.O. 4



P.O. 5

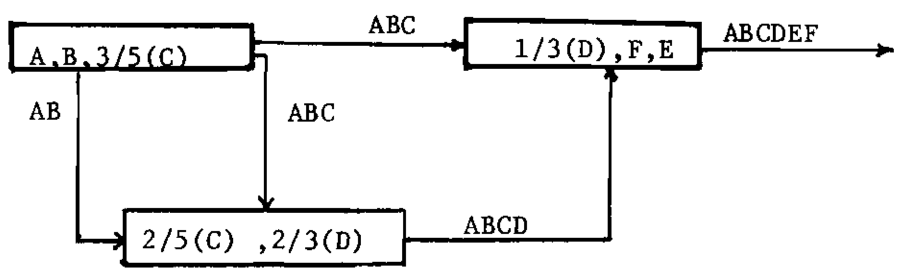


Figure 8. (Continued)

over P.O. 4 because of the same reasons for which he prefers P.O. 1 over P.O. 2.

Continuing with the example, the designer by ordinal comparisons has been able to choose one P.O. from each of the subsets of patterns. Now he must choose between P.O. 1 and P.O. 3. If the designer is willing to favor the advantages of P.O. 1 (less duplication of facilities and lower wages) over the advantages of P.O. 3 (less variety in the production stage of the units being handled and less supervision and control required), then he would choose P.O. 1 over P.O. 3. Otherwise he may choose P.O. 3 over P. O. 1 or remain undecided.

Detailed Annual Equivalent Cost Analysis

If the screening does not result in one selected P.O., a more structured approach is needed. One way to approach the problem is to estimate an equivalent annual cost after taxes for each of the remaining P.O. The equivalent annual cost for each of the "costs of paralleling" would be estimated for each P.O. (at least for the relevant costs in the specific situation). A final figure for each P.O. would be obtained by adding up the figures representing each cost. The minimum of those final figures would be taken as a valid indicator of the best P.O. Unfortunately, estimating costs 7, 8 and 9 (see pages 35-36) would be difficult in most cases. The cost of the buffer stocks is also difficult to assess, because it depends on the way the stations will be operated. The designer at this stage should account for the minimum of buffer stocks that would guarantee an adequate operation.

Cost Rating

Another simpler method to approach the problem of choosing a final P.O. is proposed. This method is based on a rating scale for each of the relevant costs. For each of those costs a value of 1 is assigned to an "artificial benchmark option" (A.B.O.), which involves "no paralleling and optimum interaction among tasks within stations" (as if the tasks could be fitted in N conventional stations with optimal interactions). Then, for each of those costs i, the P.O. yielding the maximum cost is identified (P.O. $\max_i$) by the designer, and an estimate is made of the ratio:

$$\alpha_i = \frac{\text{Cost}_i \text{ P.O. max}_i}{\text{Cost}_i \text{ A.B.O.}}$$

for each cost i. For each cost i, the other P.O.(s) are rated in the range (1, α_i) using the designer's judgment and/or available data. Now, for each specific situation, some types of costs are more significant than others in determining the total costs. Because of this, different weights should be assigned to each cost and a scale should be provided showing the relative importance of each cost with respect to the others. One way of constructing that scale is to assign 100 to the most important type of cost, the one whose total amount in equivalent annual after tax dollars is estimated to be highest. For example, the annual equivalent for wages may be judged higher than the annual equivalent of any other cost, and 100 would be assigned to wage costs. The other costs are assigned numbers that reflect the percentage amount of these costs with respect to the highest cost. The final score for each P.O. is obtained by multiplying its rating for each cost i by the relative weight of that cost and adding up over all costs. The selected P.O.

will be the one with the minimum score.

In the last example suppose that the designer has not been able to decide between P.O. 1 and P.O. 3. Table 2 demonstrates how this method could be applied.

The advantage of this method is that it does not require highly precise cost estimates to arrive to a solution. Instead, it requires:

a. An estimate of the relative importance of each type of cost with respect to the others,

b. The ideal model of an "artificial benchmark option," A.B.O.

c. For each type of cost i, identification of the "max. cost P.O." and estimation of α_i as a measure of the degree of complexity of this P.O. with respect to that cost, and related to the A.B.O. and,

d. For each cost i, a rating of all P.O. in the interval $(1, \alpha_i)$.

Cost Comparisons Between N-Cycle Options Arranged From the Same N-Cycle Fit

Given an N-cycle fit which is to be arranged in N stations (conventional and/or non-conventional), several feasible N-cycle options may be considered.

One of the heuristics used in this research is to assume that given an N-cycle fit, the feasible N-cycle options with the minimum number of non-conventional stations will dominate the other N-cycle options in terms of minimum costs.

Still, the number of dominant N-cycle options may be more than

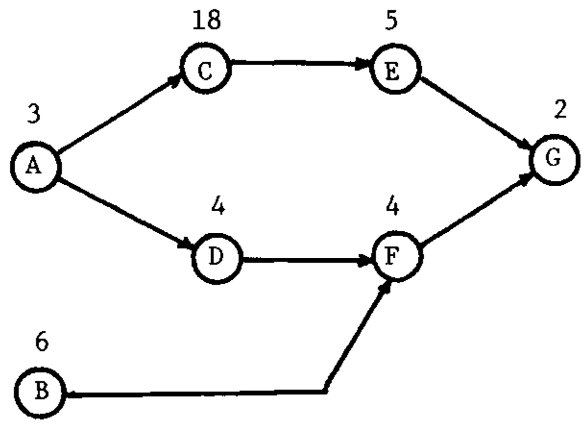
Table 2. Cost Rating of Paralleling Options

	Prod. Equip.	M.H. Equip.	Buffer Stocks	Space	Wages	Train.	Superv.	Ind. Costs	Others	Score
P.O. 1	1.2	2.5	2	1.2	1.2	2	2	1.2	-	507
P.O. 3	2.5	2	1.2	1.2	1.5	2	1.5	1.5	-	573.5
A.B.O.	1	1	1	1	1	1	1	1	1	335
Weights	65	50	30	50	100	10	10	20	-	

one. The methods proposed in the last section can also be used to compare between such N-cycle options. Consider the example of Figure 9. The designer is considering 3-cycle options. Two of the solutions the designer is considering feasible are shown. The designer could use screening, cost rating, and/or periodic cost equivalents to choose one from the set of feasible dominant 3-cycle options.

Summary

Given an N-cycle fit to be arranged in N stations, several different N-cycle options may be feasible. To select one of those N-cycle options, a heuristic rule, a screening approach, a detailed cost analysis method, and a cost rating method are proposed. The heuristic rule eliminates from further consideration those N-cycle options involving more than the minimum number of non-conventional stations. The screening approach eliminates those remaining N-cycle options which are clearly inferior to others in terms of costs. The detailed annual equivalent cost method can be used to choose one final N-cycle option provided the data on the relevant costs for each option could be estimated. The cost rating method provides for a final selection of an N-cycle option while lessening the data requirements.

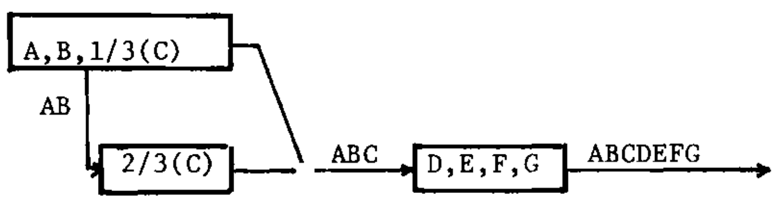


Cycle time = 15

The designer is considering 3-cycle options

Two solutions:

Solution I (1 conventional station)



Solution II (1 conventional station)

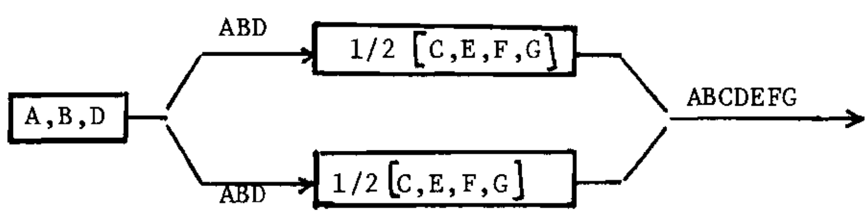


Figure 9. Two 3-Cycle Options.

CHAPTER V

A HEURISTIC METHOD OF LINE BALANCING WITH PARALLELING

Approach

The concepts and methods presented in previous chapters serve as a base to develop a heuristic method for line balancing with paralleling. This method can be considered an enlargement of the Kilbridge and Wester method of line balancing. First, a brief review of the Kilbridge and Wester method is presented. Then the basic steps of the enlarged method to include paralleling are shown using a logic flow diagram. Further explanation and refinements of the basic steps are presented. A 21 task precedence diagram is used to illustrate the application of the method.

Review of the Kilbridge and Wester Method

In 1961, Kilbridge and Wester proposed a heuristic method for line balancing. Their method has the advantage of simplicity. No computer is needed.

In their approach, the cycle time is defined as the maximum operation time the product spends at each work station. As a consequence of this definition of cycle time, the range of possible cycle times is $t_{max} \leq c \leq \Sigma t_i$; that is, the cycle time equals or exceeds the maximum task time t_{max} without exceeding the total work content time.

Having stated this, their emphasis is on minimizing the balance delay (proportion of idle time on the line due to the imperfect division of work between stations). Mathematically the balance delay is expressed as:

$$d = 100 \times \frac{nc-\Sigma t_i}{nc}$$

where $\underline{c}$ is the cycle time and $\underline{n}$ is the number of work stations (they assume one operator at each work station). They show in their article[46] how to plot a graph of minimum theoretical balance delay versus cycle time and how it could be used by management to consider alternatives differing in output and balance delay.

For line balancing, the precedence diagram is constructed as described by Jackson[47]. First, in column I of the diagram are listed all tasks which have no predecessors. Then in column K ($K \geq II$) are entered all those tasks for which all predecessor tasks are already in the diagram. While entering tasks arrows are drawn from predecessors in columns I, II, III, ..., K-1 to the tasks that must follow them in column K except when there is already a path between a predecessor and one of its followers in column K.

As a result of the way the precedence diagram has been drawn, the tasks within each vertical column are mutually independent (they are not connected by arrows) and therefore can be permuted among themselves in any sequence without violating precedence relationships. Furthermore, many tasks can be moved laterally from their columns to positions in higher numbered columns without disturbing the precedence restrictions. These two properties of the tasks in the precedence diagram permutability within columns and lateral transferability - are exploited in the attempt to obtain an optimal balance according to the objective of minimizing balance delay. A tool in their procedure is the construction of a table containing detailed information about each task taken from successive columns of the precedence diagram.

In their article[48] Kilbridge and Wester show with an example how to use permutability within columns and lateral transferability between columns to try to obtain a minimum balance delay (the reader is referred to their article if not familiar with the procedure). The authors intended for their method to be used together with "judgment and intuition" in selecting tasks within columns and transferring between columns.

The Enlarged Method

Objective and Scope

An important restriction in the K-W method is that the cycle time must be greater or equal to the time of the longest task. No provision is given in the method for the creation of stations with cycle time longer than c. The concept of paralleling of tasks and stations is not mentioned in their article.

The K-W method is appropriate for production lines where individual tasks are very small compared to total job time and also in lines where paralleling is not attactive to improve the efficiency of the balance because of difficulties in work flow.

In unpaced lines with buffer stocks between stations, continuous flow is not, in general, a prerequisite. These lines are likely to present fewer layout and operating difficulties with paralleling as opposed to the case of mechanically paced lines.

In this chapter a method of line balancing with paralleling is presented which would incorporate the alternatives of paralleling of tasks and paralleling of stations by:

1. Using tools similar to those developed by K-W.
2. Using the properties of permutability within columns and lateral transferability between columns.
3. Using the concepts, definitions, and principles of paralleling, as well as the methods for cost comparisons between options, developed in previous chapters.

The enlarged method will be called "KWP," and it may be applied in cases where there are tasks longer than the cycle time, as well as when the cycle time is such that $t_{max} \leq c$.

For a given cycle time, the KWP searches heuristically for a balance (paralleling allowed) with the approach of minimizing costs given a maximum allowable number of stations.

<u>Distribution of an N-Cycle Fit</u>

Before the KWP is presented, it is necessary to introduce some additional concepts: Given an N-cycle fit (with $N \geq 2$) it may be possible to "distribute" its tasks in two or more smaller N'_i-cycle fits where N'_i may range $1 \leq N'_i < N$, and $\Sigma N'_i = N$. (Precedence restrictions must be considered when distributing an N-cycle fit, so that any of the resulting fits may be performed in at least one sequence provided the tasks of previous fits were to be assigned previously). Each possible way of distributing a specific N-cycle fit is called a *distribution* of that N-cycle fit. The following example illustrates this concept:

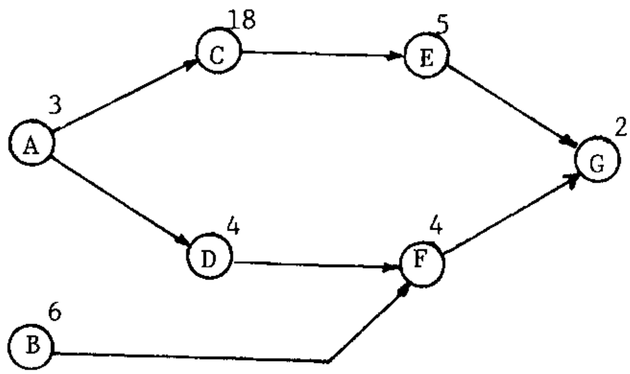


Figure 10. Distributing a 3-cycle fit.

$\{A,B,C,D,E,F,G\}$ is a 3-cycle fit

$\{A,B,D\}$ - $\{C,E,F,G\}$ is a distribution of the tasks of $\{A,B,C,D,E,F,G\}$ into one 1-cycle fit $\{A,B,D\}$ followed by one 2-cycle fit $\{C,E,F,G\}$.

$\{A,B,C\}$ - $\{D,E,F,G\}$ is a distribution of the tasks of $\{A,B,C,D,E,F,G\}$ into one 2-cycle fit $\{A,B,C\}$ followed by one 1-cycle fit $\{D,E,F,G\}$.

There appear to be $2^{N-1}-1$ different distribution patterns in which it could be possible to distribute an N-cycle fit: For a 4-cycle fit the distribution patterns are $2^{4-1}-1 = 7$

c-c-c-c	2c-2c
2c-c-c	3c-c
c-2c-c	c-3c
c-c-2c	

For a 5-cycle fit the distributed patterns are $2^{5-1}-1 = 15$

c-c-c-c-c	c-c-2c-c	2c-c-2c
2c-c-c-c	c-c-c-2c	c-2c-2c
c-2c-c-c	2c-2c-c	3c-c-c

c-3c-c	2c-3c
c-c-3c	4c-c
3c-2c	c-4c

The number of distribution patterns for N = 2 and 3 are 1 and 3 respectively.

Given a distribution of N-cycle fit, if none of the resulting N'_i cycle fits may be further distributed, then this distribution will be called a *semi-dominant distribution* of that fit. It may be possible to generate several semi-dominant distributions from one N-cycle fit. A *dominant distribution* of an N-cycle fit is defined as any semi-dominant distribution of that fit for which $\sum_{N'_i \geq 2} N'_i$ is minimum. A dominant distribution would lead ultimately to a dominant N-cycle option (if feasible), each $N'_i \geq 2$ fit being arranged in N'_i non-conventional stations. The number $\sum_{N'_i \geq 2} N'_i$ will be called the *order* of the distribution. For example, the order of a 2c-c-2c distribution is 4.

Basic Steps of the KWP

The KWP approach is briefly explained in this paragraph and is summarized in a logic flow diagram on the following pages. At each balancing iteration 1-cycle fits which "pass" a criterion of maximum allowable percentage idle time are sought. If no acceptable 1-cycle fit exists, 2-cycle fits are sought and so on. Once an acceptable N-cycle fit (N = 1,2,...,MAXN) is found, other N-cycle fits (same N) are sought in that iteration, but no higher-cycle fits are sought. For each of the acceptable N-cycle fits, dominant distributions are obtained (when a distribution is possible). Only the N-cycle fits with a minimum order of their dominant distributions are kept. The others, including those which cannot be distributed, are eliminated. In the case when no distribution of any of the N-cycle fits is possible, all the fits will be kept. From the N-cycle fits with minimum order dominant distributions, feasible N-cycle options with a number of non-conventional stations equal to the order of the dominant distributions are arranged. The cost methods presented in Chapter IV are used to select one N-cycle option from each remaining N-cycle fit. Among these N-cycle options one is finally chosen using a ratio of a cost measure divided by the sum of the task times involved in the option. The option with the minimum ratio is selected.

The flow diagram in Figure 11 shows the basic steps of the method.

Further explanations and refinements are needed relative to the following labelled steps of the logic flow diagram:

B3 - The maximum percent idle time criterion is a function of the maximum allowable number of stations for the cycle time chosen:

$$\max \% \text{ I.T.} = \frac{(S_{max} \cdot c) - \Sigma t_i}{S_{max} \cdot c} \times 100$$

where S_{max} is the maximum allowable number of stations, c is the cycle time and Σt_i is the total work content of the entire line.

B4 - It is convenient to limit the search for acceptable N-cycle fits to a maximum MAXN. The main reason for this is to limit the paralleling complications that may arise from "forcing" a set of tasks into N stations. MAXN should be greater or equal to the

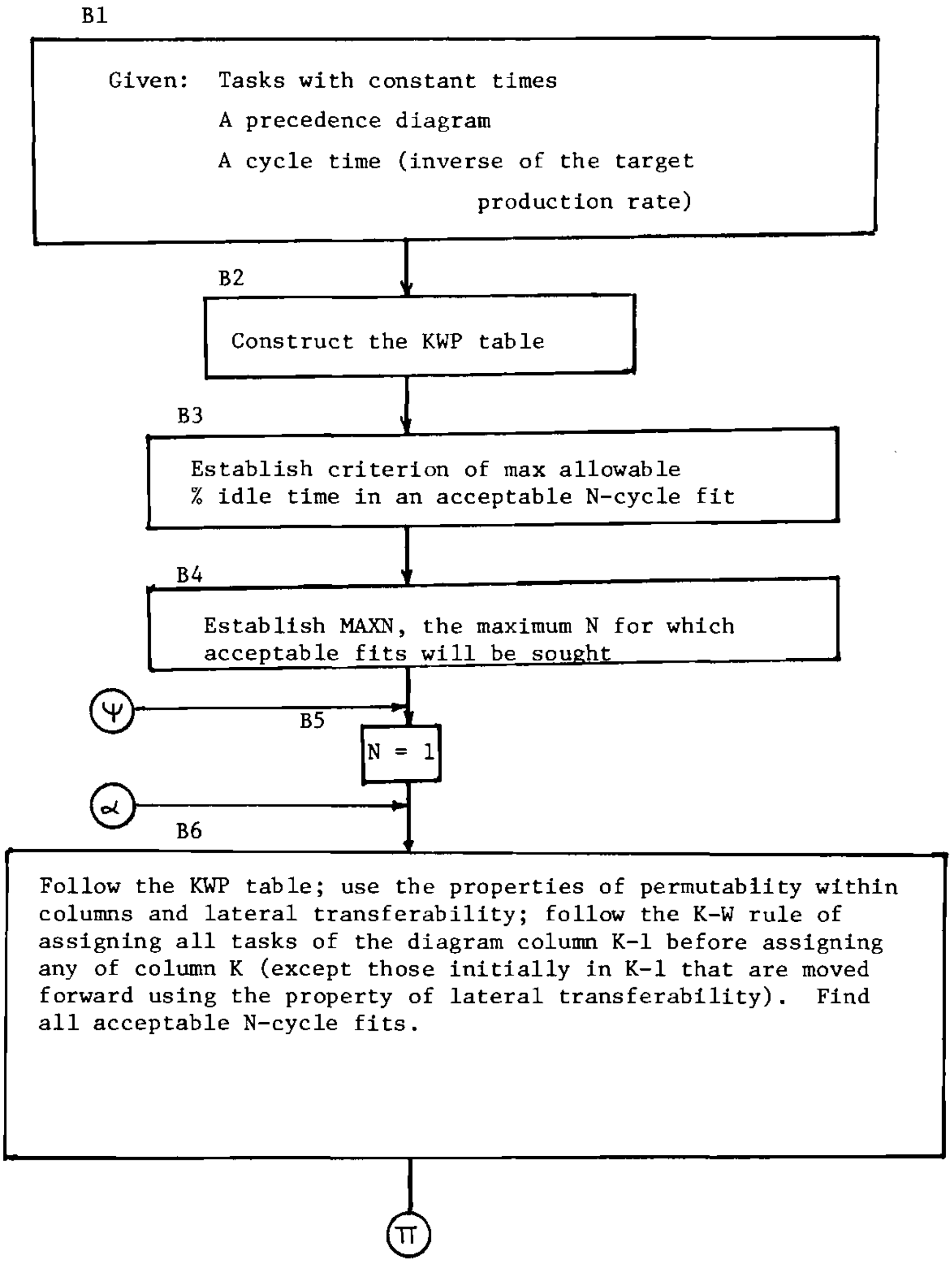


Figure 11. The Basic Steps of the KWP.

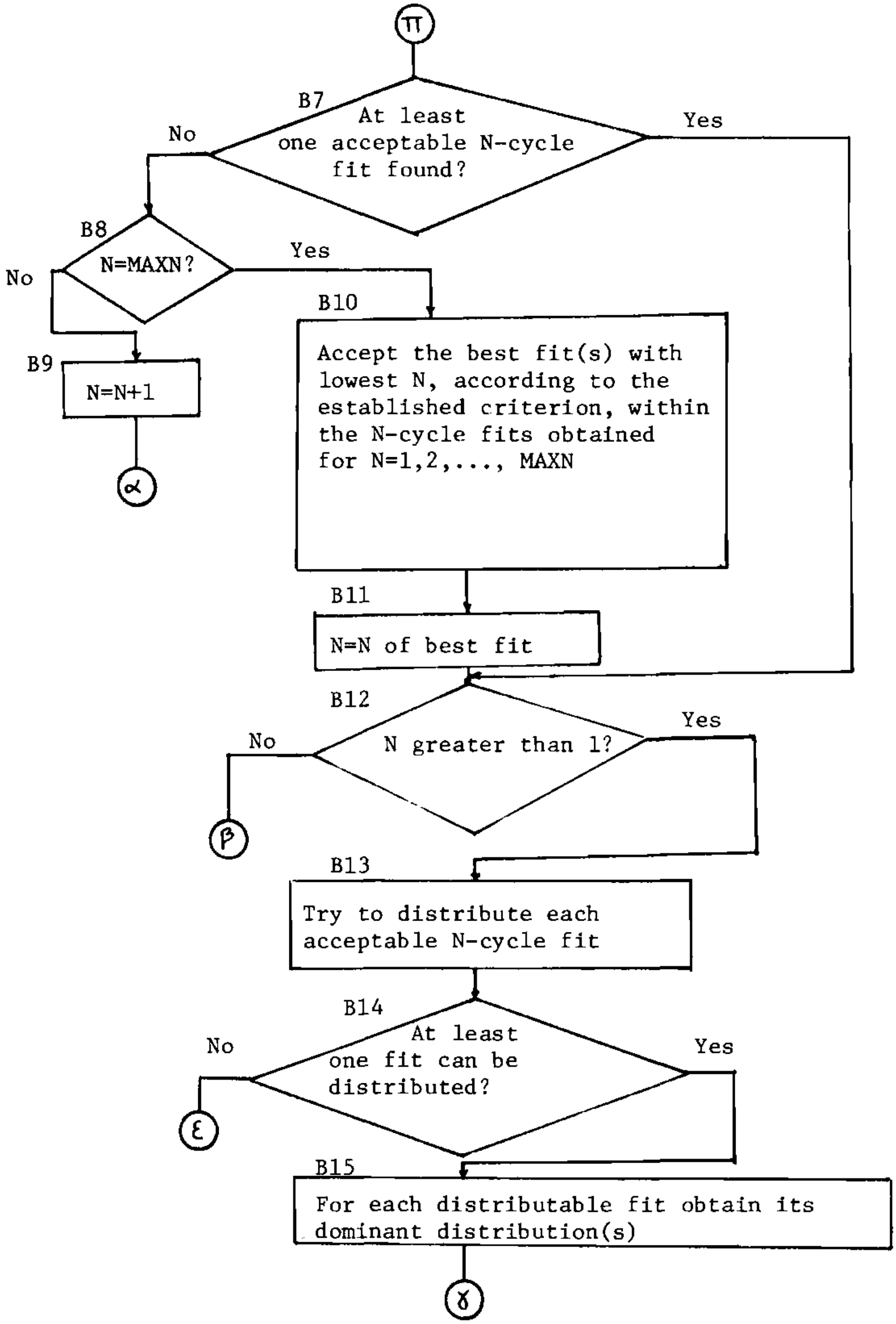


Figure 11. (Continued).

γ

B16

Eliminate those N-cycle fits with higher order of their dominant distribution(s) and those that cannot be distributed

B17

Based on their dominant distributions, for each of the remaining N-cycle fits, consider feasible N-cycle options. Each of these fits is distributed in 1-cycle fits and/or N_i'-cycle fits ($N_i' \geq 2$). Any 1-cycle fit is to be arranged in a conventional station. Any N_i'-cycle fit is to be arranged in a "simple" N_i'-cycle paralleling option with N_i' non-conventional stations

B18

For each one of the remaining N-cycle fits, select one N-cycle option using the cost comparison methods

β

B19

Obtain a final N-cycle option by choosing the one with

$$\text{Min } \frac{\text{cost measure}}{\Sigma\, t_i}$$

and/or by mere ordinal comparisons

Ø

Figure 11. (Continued).

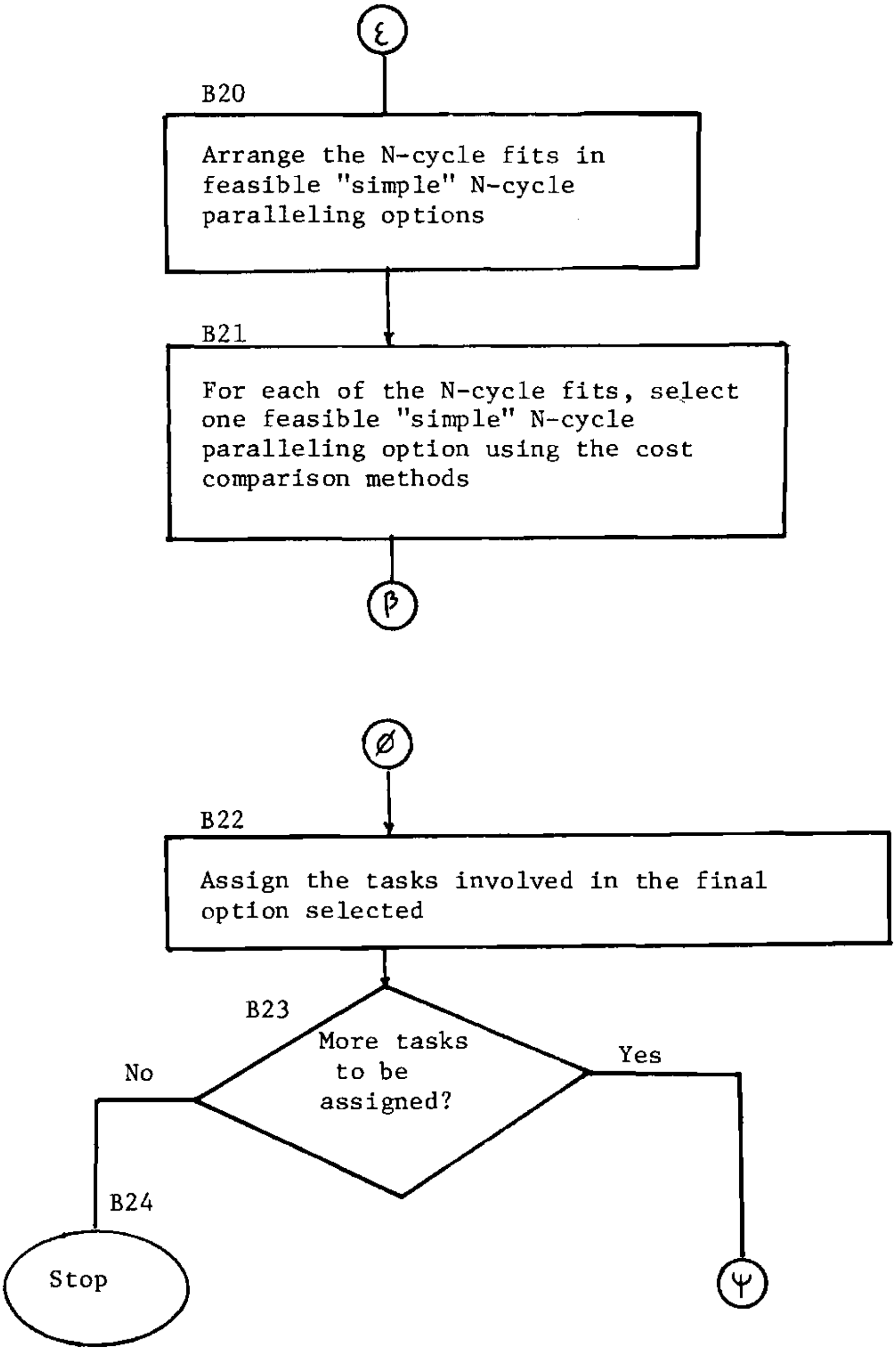


Figure 11. (Concluded).

minimum number of stations required to perform the longest task for the desired cycle time (in most practical applications MAXN should probably be 3, 4 or 5). As a consequence of this, a balance with more stations than the maximum allowable might result when the criterion cannot be met in one or more iterations.

B10 -If no acceptable N-cycle fits are found in the range (1-MAXN) the fit(s) with minimum $\frac{N \cdot c - \Sigma ti}{N \cdot c}$ found in that range will be labelled "acceptable." If several fits are tied for minimum $\frac{N \cdot c - \Sigma t}{N \cdot c}$ choose those for which N is lowest.

B13 -Once a fit is found to be acceptable, its distribution in smaller fits is done based on the precedence restrictions of the tasks in the fit without necessarily following the K-W rule of columns.

B13, B14, B15 and B16 - The conceptual steps are presented in the diagram; though, in practice the easiest way to deal with these steps is as follows: Take one of the acceptable N-cycle fits and try to distribute it according to the $2^{N-1}-1$ patterns, starting with those patterns leading to lower order dominant distributions. For Example, if a 4-cycle fit is being distributed try:

c-c-c-c	If a distribution is found, the order of the dominant distribution is zero. If not, try:
2c-c-c c-2c-c c-c-2c	If a distribution is found, the order of the dominant distribution is two. If not, try:
3c-c c-3c	If a distribution is found, the order of the dominant distribution is three. If not, try:

2c-2c If a distribution is found, the order of the dominant distribution is four. If not, it is an undistributable 4-cycle fit.

Once a distribution is found no distribution of higher order is sought, although additional distributions of the same order are sought to obtain the complete set mentioned in B15.

When distributing another acceptable N-cycle fit, there is no need to find its dominant distributions when it becomes clear that the order of its dominant distribution(s) is higher than the order of the dominant distribution(s) of a previously analyzed acceptable N-cycle fit.

In this way, the set of remaining N-cycle fits for B17 is obtained with less computational effort.

The heuristic behind this is simple: consider only those fits leading to N-cycle options with the lowest number of non-conventional stations.

B17 and B20 - At this point N-cycle options are being considered. It is the culmination of a chain of concepts: N-cycle fits, distribution patterns, distributions, paralleling option patterns, and paralleling options. Figure 12 shows how all these are linked together leading to N-cycle options. The KWP considers only a subset of set III, that subset leading to the lowest number of non-conventional stations. Cost methods are used to choose one final N-cycle option from that subset.

B19 - The methods developed in Chapter IV were intended to be used when comparing N-cycle options arranged from one N-cycle fit. In that

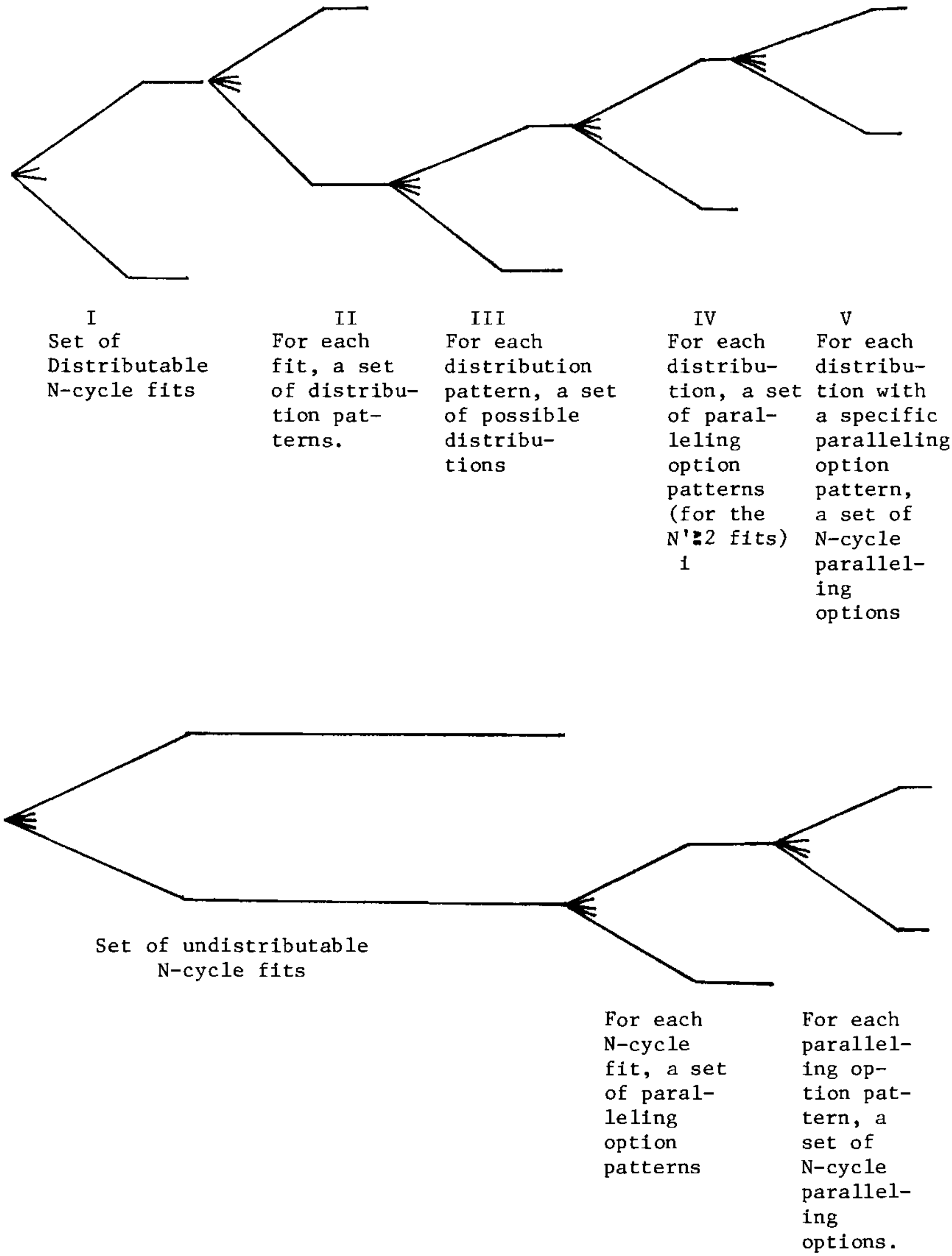


Figure 12. From N-Cycle Fits to N-Cycle Options.

case the problem is: What is the best way to arrange a specific set of tasks in N stations? When comparing between N-cycle options arranged from different N-cycle fits, the problem has another dimension; not only how to allocate a set of tasks in N stations, but also, which tasks to allocate? What is meant in B19 by "ordinal comparisons" is that the designer may clearly prefer an N-cycle option over another because of the set of tasks involved in each (that is, he may prefer to combine A,B,C and D over combining A,B,C,E and F in three stations). For the options among which the designer cannot discriminate based on clear preference, he would choose the one with minimum $\frac{\text{cost measure}}{\Sigma t_i}$, where the cost measure is either the annual equivalent cost or the cost rating score. When the cost rating score is used, it is implicitely assumed that the "artificial benchmark option" is equivalent for all the options.

Finally, one word of caution with respect to feasibility. There is no guarantee that at any iteration, feasible final N-cycle options would be obtained (feasible with respected to restrictions such as positional restrictions, groupings, etc.). For example, suppose that B17 is entered with only one 4-cycle fit with one dominant distribution of order 2, and no feasible way or arranging the tasks of the 2-cycle fit in a 2-cycle paralleling option is found. In such a case, go back to B13 and try again with the same acceptable 4 cycle fits, but this time entering B16 with 4-cycle fits with dominant distributions of order 3 (if any). In general, if feasibility is not achieved in at least one of the final N-cycle options then, the procedure would be to backtrack to some previous step and start from there again taking into account what has been learned from the unfeasible options. However, one thing that the designer can do in advance is to account for the feasibility of combining the tasks of the 1-cycle fits in conventional stations. That is, if a distribution with pattern c-c-2c is entered in B17, the designer would have accounted at least for the feasibility of the conventional stations foreseen.

An Example of the Application of the Steps

Mitchell's precedence relations for his 21 task example[49] are used for this example. (A real application in which costs can be handled explicitly will be presented in the next chapter.) The cycle time for the example was set arbitrarily at 15. Task times were assigned randomly to the 21 tasks of the example. The distribution of the task times in ranges was set *a priori* as follows:

	15	tasks	with	$0<t\leq 15$
	2	"	"	$15<t\leq 30$
	2	"	"	$30<t\leq 45$
	2	"	"	$45<t\leq 60$
Total	21			

The procedure for randomization was done as follows: Five boxes with numbers were prepared

Box 1	21	numbers	from	1 to 21
Box 2	15	"	"	1 to 15
Box 3	15	"	"	16 to 30
Box 4	15	"	"	31 to 45
Box 5	15	"	"	46 to 60

The following sampling process was followed: One task time value was drawn from box 2 and assigned to a task number drawn later from box 1. This continued until 15 numbers had been drawn from each box. (Sampling from box 2 was with replacement, whereas sampling from box 1 was without replacement.) The same procedure was used to drawn two task times from box 3 and assign them to two task numbers drawn from box 1. The same was done with boxes 4 and 5 drawing two numbers from each. Sampling with replacement was used also at boxes 3, 4 and 5. The result of the randomization of task times is shown together with the precedence diagram in Figure 13. The necessary portions of the Kilbridge and Wester table (the KWP table) is shown in Table 3.

For this example a balance was first obtained using the Kilbridge and Wester method enlarged with the heuristics proposed by Arcus[50] for dealing with tasks longer than the cycle time. No cost trade-offs were analyzed. The objective was to minimize the number of stations for a given cycle time as in conventional methods. A 27 station balance was obtained. For a description of this method see Appendix.

Suppose the designer is considering the application of the KWP to this problem. Further, suppose that he wants a balance with at most 26 stations to compare it with his previous 27 station balance (the minimum theoretical number of stations is $\frac{\Sigma t}{c} = \frac{360}{15} = 24$ with I.T. = 0). For 26 stations the expected percentage idle time per station is

$$\text{EITPS} = \frac{26\text{x}15 - 360}{26 \text{ x } 15} \text{ x } 100 = 7.69\%$$

which leads to the following acceptability criteria:

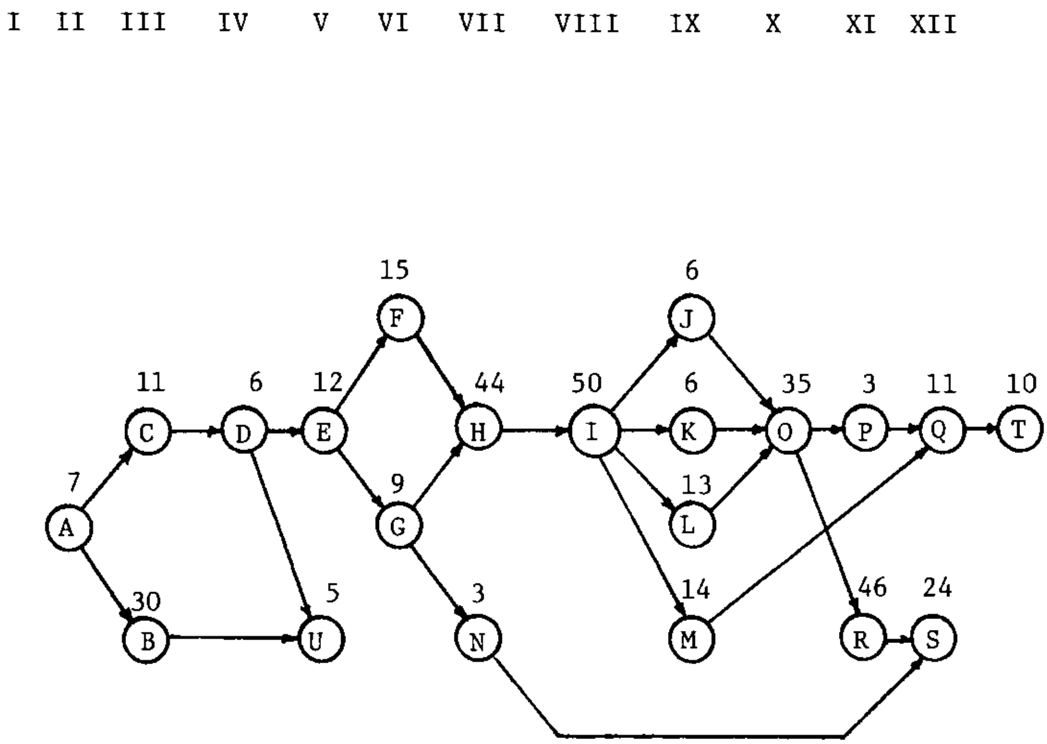


Figure 13. A 21 Task Precedence Diagram.

Table 3. The KWP Table for the 21 Task Example

Column No.	Task	Task Time	Remarks
I	A	7	
II	B	30	→ XI (U)
	C	11	
III	D	6	
IV	E	12	
	U	5	→ XII
V	F	15	
	G	9	
VI	H	44	
	N	3	→ XI (S)
VII	I	50	
VIII	J	6	
	K	6	
	L	13	
	M	14	→ X (R,S)
IX	O	35	
X	P	3	
	R	46	→ XI (S)
XI	Q	11	
	S	24	→ XII
XII	T	10	

1 cycle fit: acceptable fit if idle time of fit is $\leq$ 0.0769x15x1=1.15

2 cycle fit: acceptable fit if idle time of fit is $\leq$ 0.0769x15x2=2.31

3 cycle fit: acceptable fit if idle time of fit is $\leq$ 0.0769x15x3=3.46

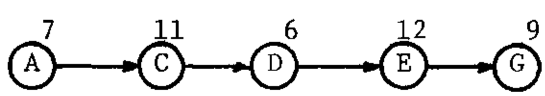
4 cycle fit: acceptable fit if idle time of fit is $\leq$ 0.769x15x4=4.61

Because of the integer values of the task times in the example the resulting numbers are rounded down for practical purposes to the nearest smaller integer.

Further, suppose the designer wants to limit the search for acceptable N-cycle fits to at most 4-cycle fits (MAXN = 4). Now he is ready to start the first iteration.

Iteration 1

The only 1-cycle fit is $\{A\}$. Transferring tasks does not provide for other fits. This fit has I.T. = 8 > 1, so it is not acceptable. By moving task B to III the best 2-cycle fit found is $\{A,C,D\}$ with I.T. = 30 - 24 = 6 > 2, and it is not acceptable. By moving task B to V and task U to XI the 3-cycle fit $\{A,C,D,E,G\}$ becomes available. This fit has I.T. = 45 - 45 = 0 < 3, so it is acceptable. No other acceptable 3-cycle fit is found. The precedence relations for $\{A,C,D,E,G\}$ are



Distributions of the fit:

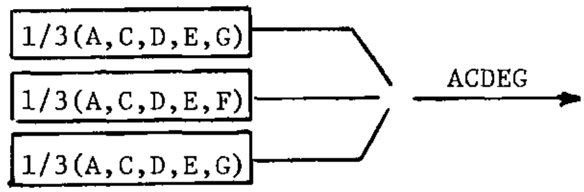
pattern c-c-c no distributions found

pattern 2c-c no distributions found

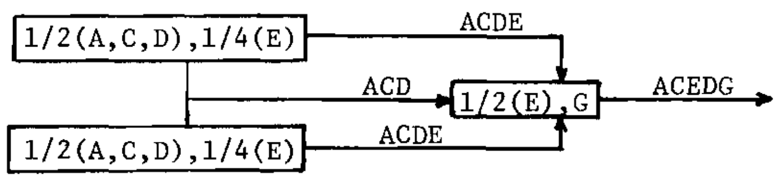
pattern c-2c no distributions found

Next, arrange the 3-cycle fit in feasible 3-cycle paralleling options. Suppose the designer after screening is left with:

P.O. 1



P.O. 2



The designer then uses one of the cost methods developed to select one of the P.O.'s. Suppose he selects P.O. 1.

Decision Iteration 1: Move tasks B to V,U to VI.

Assign tasks $\{A,C,D,E,G\}$ to P.O. 1.

Stations so far assigned: 3 Idle time carried: 0

After Iteration 1, the situation in the KWP table is shown in Table 4.

Iteration 2

The only 1-cycle fit is $\{F\}$. It has I.T. = 0, so it is acceptable. No other acceptable 1-cycle fit is found. Transferring tasks does not provide for other 1-cycle fits.

In this case there is only one 1-cycle option and it is chosen.

Decision Iteration 2: assign task F to a conventional station.

Table 4. The KWP Table for the 21 Task Example After Iteration 1

Column No.	Task	Task Time	Remarks
I	A	~~7~~	
II	~~B~~	~~30~~	~~→ XI (U)~~
	C	~~11~~	
III	D	~~6~~	
IV	E	~~12~~	
	~~U~~	~~5~~	~~→ XII~~
V	F	15	
	G	~~9~~	
	B	30	→ XI (U)
VI	H	44	
	N	3	→ XI (S)
	U	5	→ XII
VII	I	50	
VIII	J	6	
	K	6	
	L	13	
	M	14	→ X (R,S)
IX	O	35	
X	P	3	
	R	46	→ XI (S)
XI	Q	11	
	S	24	→ XII
XII	T	10	

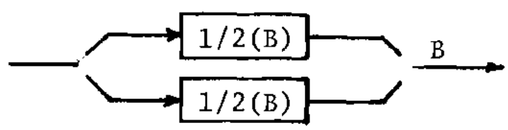
Stations so far assigned: 4. Idle time carried: 0.

Iteration 3

The best 1-cycle fit, obtained by transferring task B to VI and task U to VII is $\{N\}$ with I.T. = 12, and it is not acceptable. The only 2-cycle fit obtained is $\{B\}$ with I.T. = 30 - 30 = 0, so it is acceptable. Transferring tasks does not provide for other 2-cycle fits.

No distribution is possible for such a 2-cycle fit.

The following P.O. is chosen:



Decision Iteration 3: assign task B to parallel stations.

Stations so far assigned: 6 Idle time carried: 0

After Iteration 3 the situation in the table is shown in Table 5.

Iteration 4

No acceptable 1-cycle fits or 2-cycle fits were found, not even by transferring tasks.

The only 3-cycle fit found was $\{H\}$ with I.T. = 45 - 44 = 1 < 3, so it is acceptable.

No distribution is possible for such a 3-cycle fit.

The following P.O. is chosen:

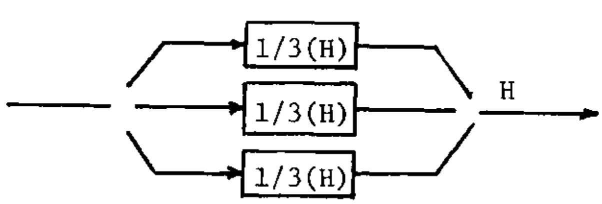


Table 5. The KWP Table for the 21 Task Example After Iteration 3

Column No.	Task	Task Time	Remarks
I	A	~~7~~	
II	~~B~~	~~30~~	~~→ XI (U)~~
	C	~~11~~	
III	D	~~6~~	
IV	E	~~12~~	
	~~U~~	~~5~~	~~→ XII~~
V	F	~~15~~	
	G	~~9~~	
	B	~~30~~	→ XI (U)
VI	H	44	
	N	3	→ XI (S)
	U	5	→ XII
VII	I	50	
VIII	J	6	
	K	6	
	L	13	
	M	14	→ X (R,S)
IX	O	35	
X	P	3	
	R	46	→ XI (S)
XI	Q	11	
	S	24	→ XII
XII	T	10	

Decision Iteration 4: assign task H to parallel stations.

Stations so far assigned: 9 Idle time carried: 1

Iteration 5

The 1-cycle fits $\{N\}$, $\{U\}$ and $\{N,U\}$ are not acceptable. No other 1-cycle fit was found.

No 2-cycle fits or 3-cycle fits were found.

The following acceptable 4-cycle fits were found:

$\{N,U,I\}$ with I.T. = 2

$\{I,J\}$ with I.T. = 4 (by moving U to VIII)

$\{I,K\}$ with I.T. = 4 (by moving U to VIII)

$\{N,I,J\}$ with I.T. = 1 (by moving U to VIII)

$\{N,I,K\}$ with I.T. = 1 (by moving U to VIII)

No distribution is possible for any of the fits because of the long task I involved in all of them.

The next steps are, based on the precedence relations:

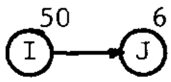
-Arrange the fits in feasible 4-cycle paralleling options.

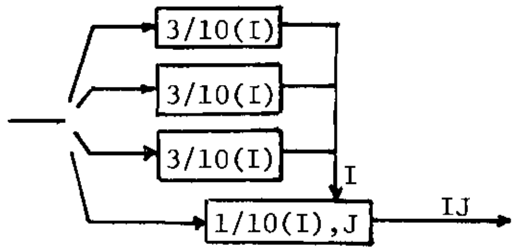
-Decide for one 4-cycle paralleling option from each fit.

-Choose one final P.O. by Min. $\frac{\text{Cost measure}}{\Sigma t_i}$ and/or ordinal comparisons.

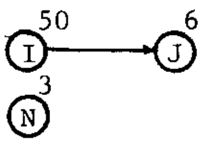
Suppose $\{N,U,I\}$ is not feasible to be arranged in a 4-cycle paralleling option and that the following paralleling options are chosen for each of the other fits:

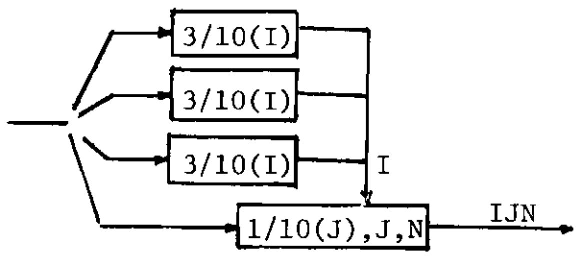
Fit $\{I,J\}$



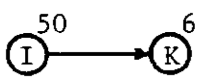


Fit $\{N,I,J\}$



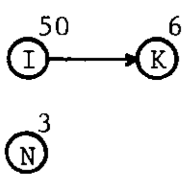


Fit $\{I,K\}$



The P.O. is similar to the one represented for the fit $\{I,J\}$

Fit $\{N,I,K\}$



The P.O. is similar to the one represented for the fit $\{N,I,J\}$

Suppose also that $\dfrac{\text{cost measure}}{\Sigma t_i}$ is minimum for the P.O. chosen for $\{N,I,K\}$

Decision Iteration 5: Move task U to VIII.

Assign tasks $\{N,I,K\}$ to the P.O. chosen.

Stations so far assigned: 13 Idle time carried: 2

After Iteration 5 the situation in the table is shown in Table 6.

Iteration 6

One acceptable 1-cycle fit {M} is found, with I.T. = 1.

The situation is similar to the situation with Iteration 2.

Decision Iteration 6: Assign task M to a conventional station

Stations so far assigned: 14 Idle time carried: 3

Iteration 7

The best 1-cycle fit L with I.T. = 2, is not acceptable.

No acceptable 2-cycle fits or 3-cycle fits were found.

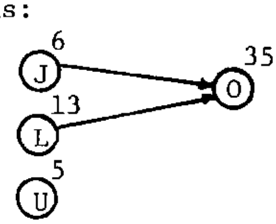
Two acceptable 4-cycle fits were found:

{J,L,U,O} with I.T. = 1

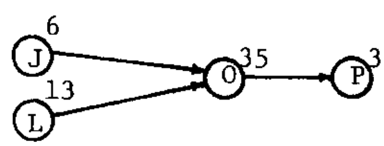
{J,L,O,P} with I.T. = 3 (by moving U to X)

The precedence relations:

{J,L,U,O}



{J,L,O,P}



Distributions for fit {J,L,O,U}

pattern c-c-c-c not possible

pattern 2c-c-c not possible

pattern c-2c-c not possible

pattern c-c-2c not possible

Table 6. The KWP Table for the 21 Task Example After Iteration 5

Column No.	Task	Task Time	Remarks
I	A	~~7~~	
II	~~B~~	~~30~~	~~→ XI (U)~~
	C	~~11~~	
III	D	~~6~~	
IV	E	~~12~~	
	~~U~~	~~5~~	~~→ XII~~
V	F	~~15~~	
	G	~~9~~	
	B	~~30~~	→ XI (U)
VI	H	~~44~~	
	N	~~3~~	→ XI (S)
	~~U~~	~~5~~	~~→ XII~~
VII	I	~~50~~	
VIII	J	6	
	K	~~6~~	
	L	13	
	M	14	→ X(R,S)
	U	5	→ XII
IX	O	35	
X	P	3	
	R	46	→ XI(S)
XI	Q	11	
	S	24	→ XII
XII	T	10	

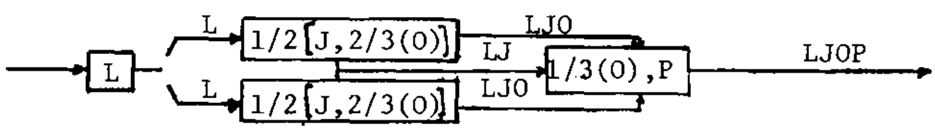
pattern 3c-c not possible

pattern c-3c not possible

pattern 2c-2c not possible

For the fit {J,L,O,P} the distribution {L} - {J,O,P} is possible (a 1-cycle fit followed by a 3-cycle fit). This is the dominant distribution (the only one) for the fit.

The next steps are to consider feasible 4-cycle options and to choose one by cost comparisons. Suppose the chosen 4-cycle option is:



Decision Iteration 7: Move U to X. Assign tasks {L,J,O,P} to the 4-cycle option above.

Stations so far assigned: 18 Idle time carried: 6

After Iteration 7 the situation in the table is shown in Table 7.

Iteration 8:

No acceptable N-cycle fits are found for N = 1,2, or 3.

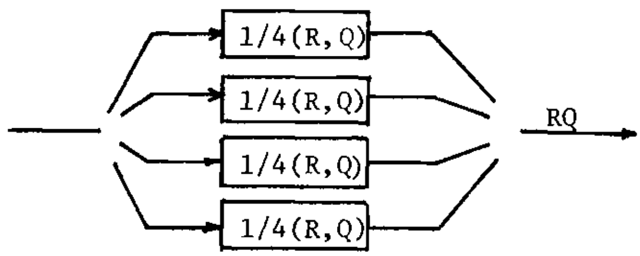
The 4-cycle fit {R,Q} with I.T. = 60 - 57 = 3 < 4 is acceptable.

To obtain this fit, it was necessary to transfer U to XI. It is the only acceptable 4-cycle fit obtained, and it cannot be distributed.

Suppose that 4 parallel stations were chosen as the final 4-cycle paralleling option:

Table 7. The KWP Table for the 21 Task Example After Iteration 7

Column No.	Task	Task Time	Remarks
I	A	~~7~~	
II	~~B~~	~~30~~	~~→ XI (U)~~
	C	~~11~~	
III	D	~~6~~	
IV	E	~~12~~	
	~~U~~	~~5~~	~~→ XII~~
V	F	~~15~~	
	G	~~9~~	
	B	~~30~~	→ XI (U)
VI	H	~~44~~	
	N	~~3~~	→ XI (S)
	~~U~~	~~5~~	~~→ XII~~
VII	I	~~50~~	
VIII	J	~~6~~	
	K	~~6~~	
	L	~~13~~	
	M	~~14~~	→ X (R,S)
	~~U~~	~~5~~	~~→ XII~~
IX	O	~~35~~	
X	P	~~3~~	
	R	46	→ XI (S)
	U	5	→ XII
XI	Q	11	
	S	24	→ XII
XII	T	10	



Decision Iteration 8: Move task U to XI. Assign R and Q to four parallel stations.

Stations so far assigned: 22 Idle time carried: 9

Iteration 9:

An acceptable 1-cycle fit is found after transferring task S to XII it is $\{U, T\}$ with I.T. = 0.

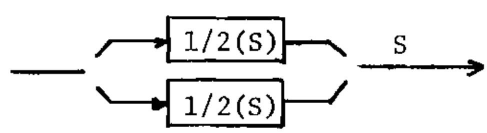
The fit is assigned to a conventional station.

Decision Iteration 9: Move task S to XII. Assign tasks U and T to a conventional station.

Stations so far assigned: 23 Idle time carried: 6

Iteration 10:

There is only one task remaining, task S. It is arranged in two parallel stations.



Decision Iteration 10: Assign task S to two parallel stations.

Stations so far assigned: 25 Idle time carried: 15

Note that the number of stations obtained (25) is one less than the target for the problem.

Summary

The objective of the method developed is to obtain a balance within a maximum allowable number of stations while tending to minimize the costs of the line. The method uses tools developed by Kilbridge and Wester for their line balancing method. At the beginning of each iteration, N-cycle fits are sought using Kilbridge and Wester's rule of columns and the properties of permutability within columns and lateral transferability.

A criterion limiting the idle time per station assigned in each iteration is stated. In order to achieve this criterion, paralleling is an allowed alternative. The cheapest way of achieving the criterion is sought, even if paralleling is necessary in order to do so.

The concepts of distributions, distribution patterns, semi-dominant distributions, dominant distributions and order were presented. The aim is to organize and guide the search for dominant N-cycle options at each iteration. Cost comparison methods and heuristics are then used to select a final option.

CHAPTER VI

APPLICATION OF THE KWP TO A REAL PROBLEM

The Product

The assembly of a Windsor side chair is used to demonstrate the method. The chair is being assembled (at the time of this research) by a company located approximately 30 miles from downtown Atlanta. The production system is an unpaced assembly line.

The parts of the chair are manufactured outside the United States and are imported for assembly. The parts may be divided in two classes: (a) Those to be incorporated in the legs assembly, and (b) Those to be incorporated in the seat assembly. Figure 14 shows the assembled product.

Process Data

This section presents the tasks to be performed with their performance times and the major equipment and tools needed for the assembly. No recorded data on either average or standard times was available. Furthermore, no standard method was being employed. The average times presented here are the result of a series of direct observations of the tasks and their measurement with a hand watch. The times are approximate, rather than the product of a formal statistical study. Table 8 presents the process data.

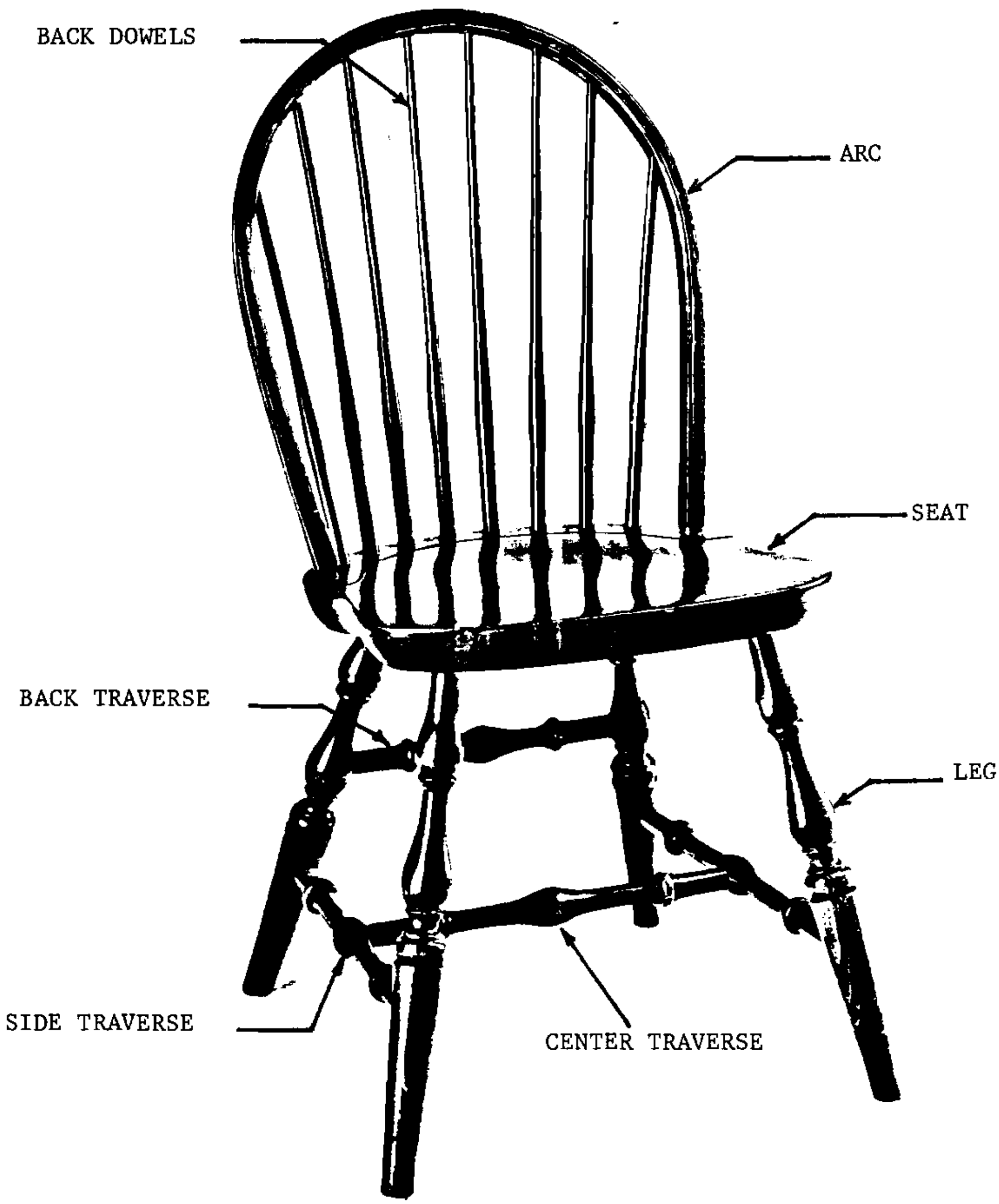


Figure 14. Windsor Side Chair.

Table 8. Process Data for the Windsor Chair

Task Symbol	Task Description	Approx. Avg. Time (sec)	Equipment
A	Glue 8 dowel holes in the seat	12	Bench
B	Assemble 8 back dowels to seat (4 pairs of different length)	33	Bench
C	Glue 8 holes of the arc	11	Bench
D1	Glue 2 arc holes in seat	5	Bench
D2	Assemble arc to seat and dowels	21	Bench
E	Press arc in seat and dowels	23	Press
F	Seal 2 holes in bottom of seat with small wood plugs	12	Stand
G	Trim wood plugs	5	Horizontal blade table
H	Stain wood plugs	5	Stand
I	Glue 4 holes in two back legs	15	Bench & Wood Support
J	Assemble 2 back legs together with back traverse	23	Bench & Wooden Frame
K	Glue 2 holes in 2 side traverses	8	Bench & Wood Support
L	Assemble 2 side traverses with center traverse	17	Bench & Wooden Frame
M	Glue holes in 2 front legs	12	Bench & Wood Support
N	Assemble piece from task L to 2 front legs	13	Bench & Wooden Frame

Table 8. (Continued)

Task Symbol	Task Description	Approx. Avg. Time (sec)	Equipment
O	Assemble the piece resulting from task J with the piece resulting from task N	13	Bench & Wooden Frame
P	Glue 4 holes in bottom of seat	11	Stand
Q	Assemble the legs sub-assembly to the seat sub-assembly	15	Stand
R	Drill 4 holes, one through each leg and penetrating the seat	14	Portable compressed air drill & Stand
S	Screw 4 screws, one in each drilled hole	26	Portable compressed air screwdriver & Stand
T	Stain the joints leg-seat	27	Stand
U	Staple the 8 joints in the legs sub-assembly	11	Automatic Stapler & Bench or Stand
V	Level the chair legs	15	Horizontal Blade Table
W	Clean-Stain-Inspect	60	Bench & Stand
X	Place coarrugated paper on chair	25	
Y	Drive 4 tacks, one in bottom of each leg	24	Stand
Z1	Set up protective carton (one per two chairs)	$\frac{20}{2} = 10$	Tape Mach.& Bench
Z2	Place protective carton in box (once per two chairs)	$\frac{10}{2} = 5$	
Z3	Place chair in box and close box (two chairs in one box)	12	Tape Mach.

The Precedence Diagram

Figure 15 shows a precedence diagram for the tasks described above. Due to the nature of some of the tasks, it is preferable for some sets of tasks to be performed at the same work station. This is the case with the tasks that comprise gluing and subsequent assembly, these are:

Symbol of the Tasks	Symbol of the Grouped Task	Approx. Avg. Time
A and B	(A,B)	45
C, D1 and D2	(C,D)	37
K and L	(K,L)	25
M and N	(M,N)	25
I, J and O	(I,J,O)	51
P and Q	(P,Q)	26

Also, it is preferable to perform W,T, and H at the same work station. The grouped task will be represented by (W,T,H) and its approximate average time will be 90 seconds. Finally, it is logical for Z2 to be performed together with Z3.

The new precedence diagram, after grouping, is shown in Figure 16.

No other absolute restrictions are stated *a priori*. It is understood, however, that working on tasks related to the two different major sub-assemblies (seat and legs) at the same work station may be very undesirable. In general for a work station, combinations of tasks performed on the same sub-assembly will be preferred over combinations of tasks involving both sub-assemblies.

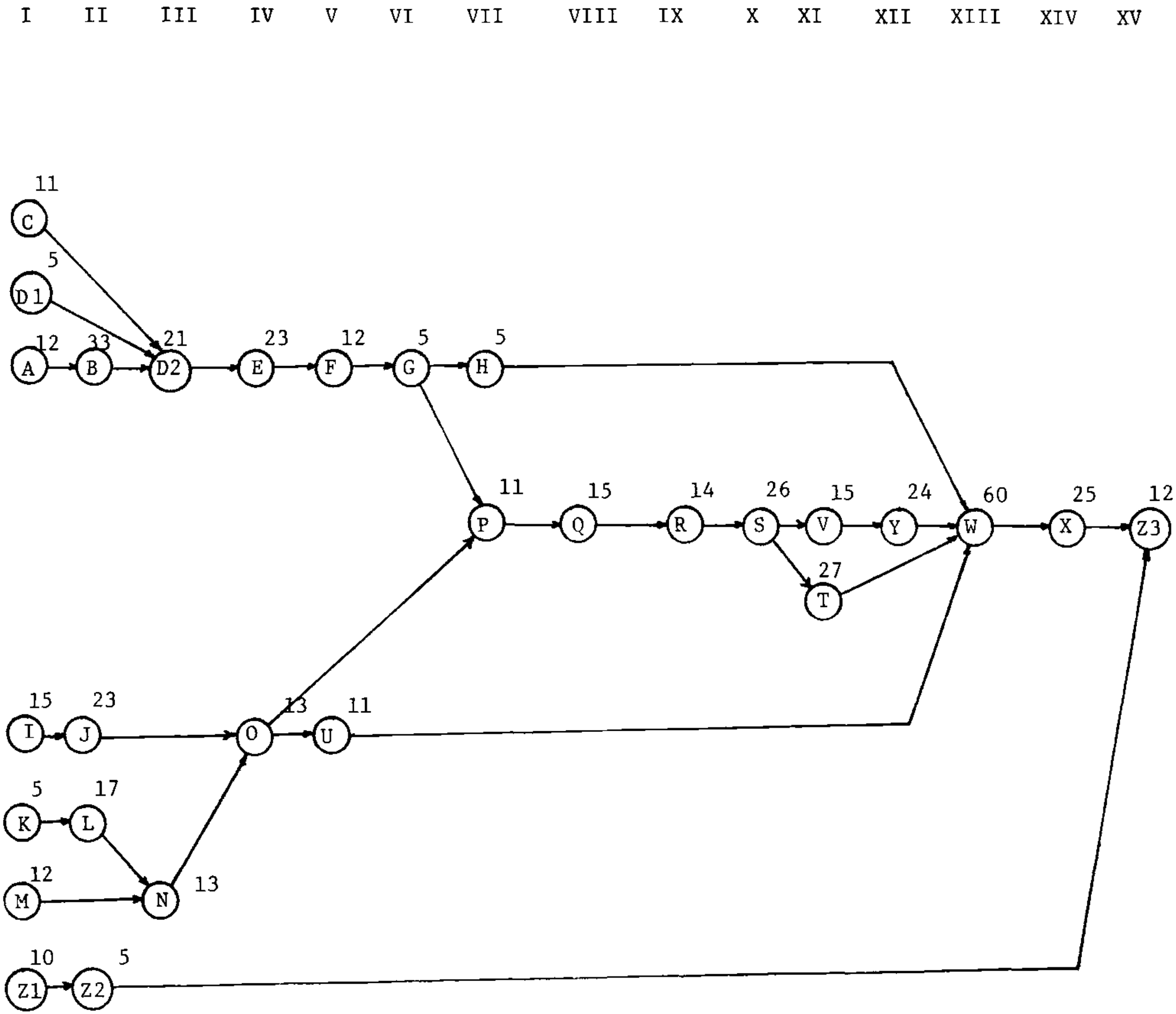


Figure 15. Precedence Diagram of the Windsor Chair (Before Grouping).

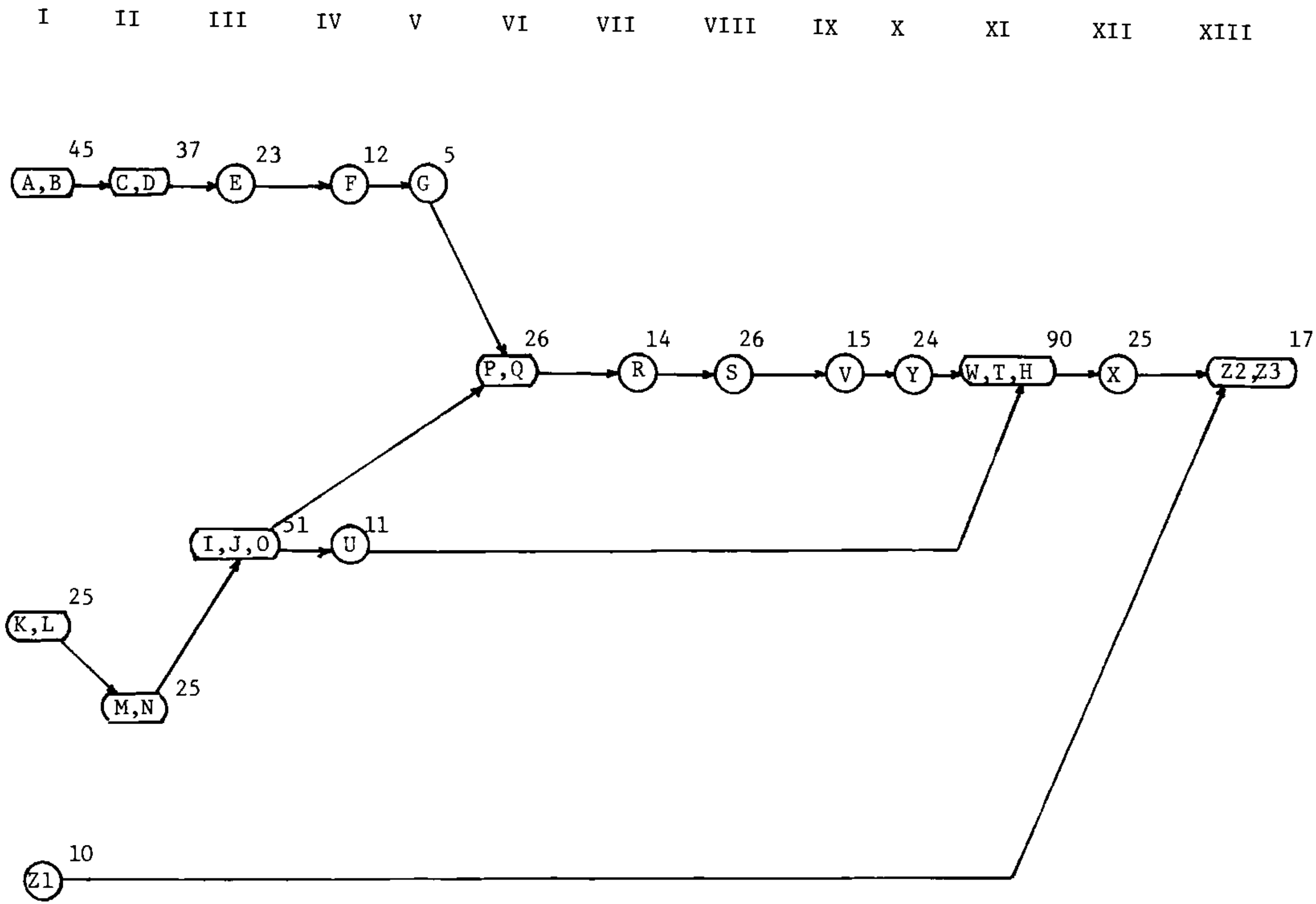


Figure 16. Precedence Diagram of the Windsor Chair (After Grouping).

The KWP Table

From the precedence diagram, the KWP table was developed as shown in Table 9.

The KWP Applied

The example will be balanced for a cycle time c = 30 seconds. With this cycle time the minimum number of stations achievable is $\frac{\Sigma t}{c} = \frac{481}{30} = 16.03$ or 17 stations.

By applying an enlargement of the Kilbridge and Wester method described in the Appendix, in which COMSOAL's heuristics for tasks longer than the cycle time are incorporated, a balance of 19 stations was obtained when a target of 18 stations was employed (18 stations represents approximately 11 percent idle time).

An allowable maximum of 18 stations was established for the application of the KWP. Based on this target, the criterion for maximum percentage idle time in an N-cycle fit is:

$$\max \% \text{ I.T.} = \frac{18x30-481}{18x30} \text{ x } 100 = 10.9\%$$

which results in the following. . .

-A 1-cycle fit is acceptable if I.T. .109x30=3.27 or 3 (sec.)

-A 2-cycle fit is acceptable if I.T. .109x60=6.54 or 6 (sec.)

-A 3-cycle fit is acceptable if I.T. .109x90=9.81 or 9 (sec.)

-A 4-cycle fit is acceptable if I.T. .109x120=13.08 or (13 (sec.)

MAXN is set equal to 4.

Iteration 1

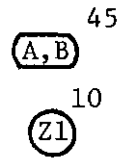
The best 1-cycle fit found is $\{K,L\}$ with I.T. = 5. It is not

Table 9. The KWP Table for the Windsor Chair

Column N.	Legs	Tasks Other	Seat	Task Times	Remarks
I			(A,B)	45	
	(K,L)			25	→III((M,N),(I,J,O),U)
		Z1		10	→XI
II			(C,D)	37	
	(M,N)			25	→IV((I,J,O),U)
III			E	23	
	(I,J,O)			51	→V(U)
IV			F	12	
		U		11	→X
V			G	5	
VI		(P,Q)		26	
VII		R		14	
VIII		S		26	
IX		V		15	
X		Y		24	
XI		(W,T,H)		90	
XII		X		25	
XIII		(Z2,Z3)		17	

acceptable. Moving tasks does not lead to other 1-cycle fits. The fit $\{(A,B),Z1\}$ is a 2-cycle fit with I.T. = 60 - 45 - 10 = 5. It is acceptable. No other acceptable 2-cycle fit is found, not even by transferring elements.

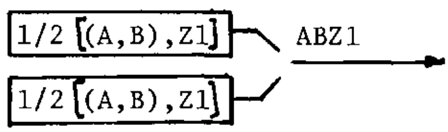
The precedence relations are:



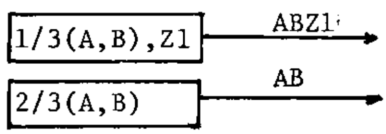
No distribution of this 2-cycle fit is possible.

Arranging the fit in feasible 2-cycle options yields:

P.O. 1



P.O. 2



Screening: There is no major difference in cost between these paralleling options. Duplicating a tape machine for Z1 seems trivial. (The possibility also exists of sharing one tape machine). P.O. 2, however, is a more complex option.

Decision Iteration 1: assign $\{(A,B),Z1\}$ to the P.O. 1 shown above.

Stations assigned so far: 2 I.T. incurred so far: 5

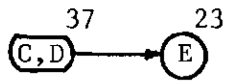
Iteration 2

The only 1-cycle fit is $\{(K,L)\}$ with I.T. = 5. It is not acceptable.

By transferring: (K,L) to III, (M,N) to IV, (I,J,O) to V, and

U to VI, an acceptable 2-cycle fit is found, it is $\{(C,D),E\}$ with I.T. = 60 - 37 - 23 = 0. No other acceptable 2-cycle fit is found.

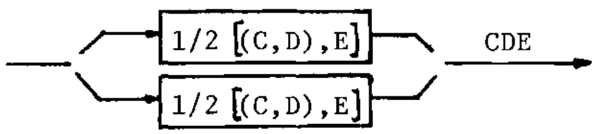
The precedence relations for the acceptable fit:



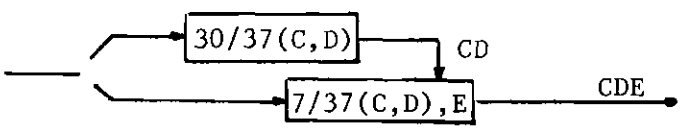
No distribution of this fit is possible.

Arranging the fit in 2-cycle paralleling options:

P.O. 1



P.O. 2



Screening: There is a high cost (approximately $2,000) for duplicating the press for E. Also, the wages paid for E are higher than those paid for (C,D) (approximately $4.60/hour versus $4 hour). These considerations are sufficient to prefer P.O. 2 over P.O. 1.

Decision Iteration 2: move (K,L) to III, (M,N) to IV, (I,J,O) to V, and U to VI. Assign $\{(C,D),E\}$ to P.O. 2 shown above.

Stations so far assigned: 4 I.T. incurred so far: 5

The situation in the KWP table is shown in Table 10.

Table 10. The KWP Table for the Windsor Chair After Iteration 2

Column N.	Legs	Tasks Other	Seat	Task Times	Remarks
I			(A,B)	~~45~~	
	~~(K,L)~~			~~25~~	→ ~~II (M,N),(I,J,O),U~~
		Z1		~~10~~	→ XI
II			(C,D)	~~37~~	
	~~(M,N)~~			~~25~~	→ ~~IV ((I,J,O),U)~~
III			E	~~23~~	
	~~(I,J,O)~~			~~51~~	→ ~~V (U)~~
	(K,L)			25	
IV			F	12	
		~~U~~		~~11~~	→ ~~X~~
	(M,N)			25	
V			G	5	
	(I,J,O)			51	
VI		(P,Q)		26	
		U		11	→ X
VII		R		14	
VIII		S		26	
IX		V		15	
X		Y		24	
XI		(W,T,H)		90	
XII		X		25	
XIII		(Z2,Z3)		17	

Iteration 3

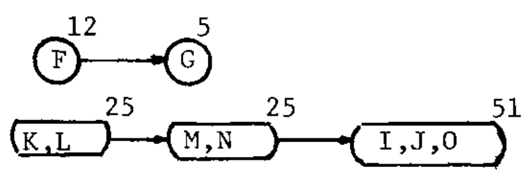
The only 1-cycle fit is $\{(K,L)\}$ with I.T. = 5. It is not acceptable.

The only 2-cycle fit is $\{(K,L),F\}$ with I.T. = 60 - 25 - 12 = 23. It is not acceptable.

The best 3-cycle fit is $\{(K,L),F,(M,N),G\}$ with I.T. = 90 - 25 - 12 - 25 - 5 = 23. It is not acceptable.

The best 4-cycle fit is $\{(K,L),F,(M,N),G,(I,J,O)\}$ with I.T. = 120 - 25 - 12 - 25 - 5 - 51 = 2. It is acceptable.

The precedence relations for the acceptable fit:



Distribution patterns for the fit:

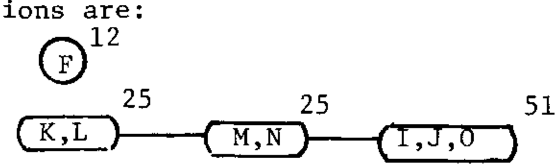
pattern c-c-c-c	no distribution is possible
pattern 2c-c-c	no distribution is possible
pattern c-2c-c	no distribution is possible
pattern c-c-2c	no distribution is possible
pattern 3c-c	no distribution is possible
pattern c-3c	no distribution is possible
pattern 2c-2c	no distribution is possible

Next is to arrange the fit in 4-cycle paralleling options. In all of these paralleling options there would be at least one station in which tasks on both the seat and legs sub-assemblies would be performed. It is preferable to avoid this, especially in the midst of a 4-cycle paralleling option. So the acceptable

4-cycle fit will be treated as infeasible.

Another acceptable 4-cycle fit is found. It is $\{(K,L), F, (M,N), (I,J,O)\}$ with I.T. = 120 - 25 = 12 = 25 = 51 = 7.

The precedence relations are:



The dominant distribution for this fit corresponds to a c-3c pattern. It is $\{(K,L)\}$ - $\{F, (M,N), (I,J,O)\}$. This distribution leads to a conventional station and a 3-cycle paralleling option where seat operations are performed together with legs operations in at least one station. No other distribution of this fit is possible. This acceptable 4-cycle fit will also be treated as infeasible.

No feasible acceptable N-cycle fit was found. The best non-acceptable N-cycle fit (best in percentage idle time) will be considered. This is the 1-cycle fit $\{(K,L)\}$ for which the percentage I.T. is $\frac{5}{30}$ = 17%.

Decision Iteration 3: Assign task (K,L) to a conventional station.

Stations assigned so far: 5 I.T. incurred so far: 10

Iteration 4

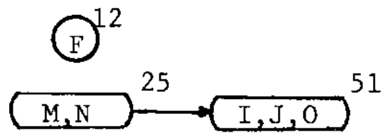
The best 1-cycle fit is $\{(M,N)\}$ with I.T. = 5. It is not acceptable.

The best 2-cycle fit is $\{(M,N), F, G\}$ with I.T. = 60 - 25 - 12 - 5 = 18.

It is not acceptable.

The only 3-cycle fit is $\{F,(M,N),(I,J,O)\}$ with I.T. = 90 - 12 - 25 - 51 = 2. It is acceptable.

The precedence relations:

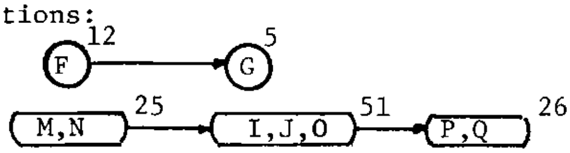


No distribution of the fit is possible.

The fit is to be arranged in 3-cycle paralleling options. In any of these options, task F (performed on the seat) must be performed with a task involving the leg sub-assembly in at least one station. It is preferable to avoid this. The acceptable 3-cycle fit will be treated as infeasible.

The best 4-cycle fit is $\{F,G,(M,N),(I,J,O),(P,Q)\}$ with I.T. = 120 - 12 - 5 -25 - 51 - 26 = 1. It is acceptable.

Precedence relations:



No distribution is possible for this 4-cycle fit.

It is preferable to avoid 4-cycle paralleling options arranged from this fit for reasons similar to those concerning the previous acceptable 3-cycle fit. So, this fit will be treated as infeasible.

considering other 4-cycle fits, the only one found is $\{F,G,(M,N),(I,J,O),U\}$ with I.T. = 120 - 12 - 5 - 25 - 51 - 11 = 16. It is not acceptable.

The best non-acceptable fit will be considered:

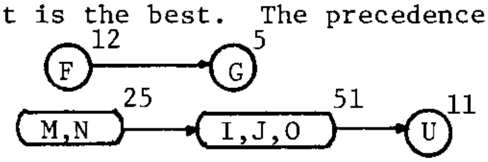
for the best 1-cycle fit %I.T. = $\frac{5}{30}$ = 17%

for the best 2-cycle fit %I.T. = $\frac{18}{60}$ = 30%

the only 3-cycle fit found was considered infeasible

for the best 4-cycle fit %I.T. = $\frac{16}{120}$ = 13%

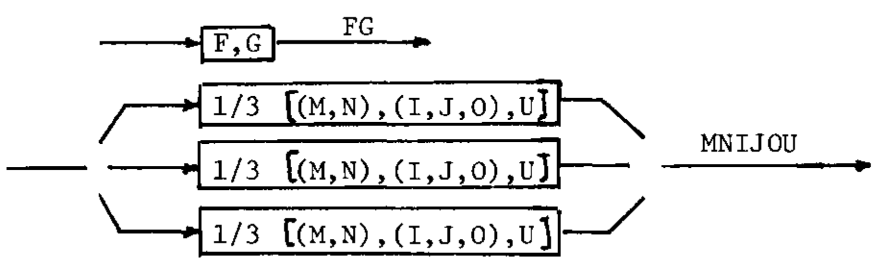
The 4-cycle fit is the best. The precedence relations are:



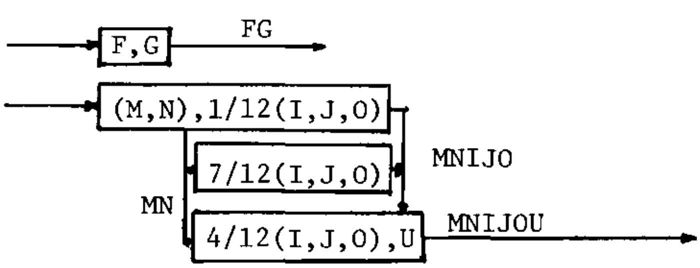
There exists a c-2c-c distritution which involves a seat-leg conventional station with tasks F,G and U being performed. This combination is not as undesirable as others in previous cases, nevertheless, in order to be consistent with the approach taken, this distribution will be avoided.

A c-3c distribution exists in which this problem does not arise. It is {F,G} - {(M,N),(I,J,O),U} . Considering feasible 4-cycle options from this distribution:

Option 1



Option 2



Screening: Low cost of facilities and equal wages for each task make Option 1 preferable to Option 2 and to other options involving complexities in buffer stocks, supervision and material flow.

Decision Iteration 4: Assign $\{F, G, (M,N), (I,J,O), U\}$ to Option 1 shown above.

Stations assigned so far: 9 I.T. incurred so far: 26

Table 11 shows the KWP table after Iteration 4.

Iteration 5

The only 1-cycle fit found is $\{(P,Q)\}$ with I.T. = 4. It is not acceptable.

The only 2-cycle fit is $\{(P,Q), R\}$ with I.T. = 60 - 26 - 14 = 20. It is not a-ceptable.

A 3-cycle fit is $\{(P,Q), R, S, V\}$ with I.T. = 90 - 26 - 14 - 26 - 15 = 9.

It is acceptable. No other acceptable 3-cycle fit is found.

The precedence relations for the acceptable fit are:

$$\overset{26}{(P,Q)} \longrightarrow \overset{14}{R} \longrightarrow \overset{26}{S} \longrightarrow \overset{15}{V}$$

Distribution patterns:

c-c-c is not possible to obtain a distribution

2c-c is not possible to obtain a distribution

c-2c a distribution exists

The distribution for the c-2c pattern is $\{(P,Q)\}$ - $\{R,S,V\}$. It is the dominant distribution. It leads to an arrangement of task (P,Q) in a conventional station and of $\{R,S,V\}$ in a

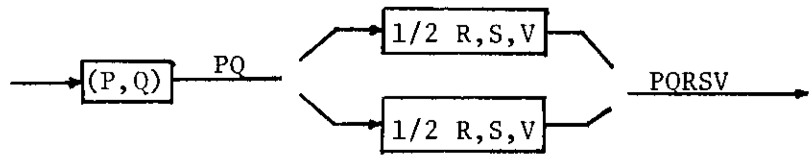
Table 11. The KWP Table for the Windsor Chair After Iteration 4

Column N.	Legs	Task Other	Seat	Task Times	Remarks
I			(A,B)	~~45~~	
	~~(K,L)~~			~~25~~	→ ~~III((M,N),(I,J,O),U)~~
		Z1		~~10~~	→ XI
II			(C,D)	~~37~~	
	~~(M,N)~~			~~25~~	→ ~~IV((I,J,O),U)~~
III			E	~~23~~	
	~~(I,J,O)~~			~~51~~	→ ~~V(U)~~
	(K,L)			~~25~~	
IV			F	~~12~~	
		~~U~~		~~11~~	→ ~~X~~
	(M,N)			~~25~~	
V			G	~~5~~	
	(I,J,O)			~~51~~	
VI		(P,Q)		26	
		U		~~11~~	→ X
VII		R		14	
VIII		S		26	
IX		V		15	
X		Y		24	
XI		(W,T,H)		90	
XII		X		25	
XIII		(Z2,Z3)		17	

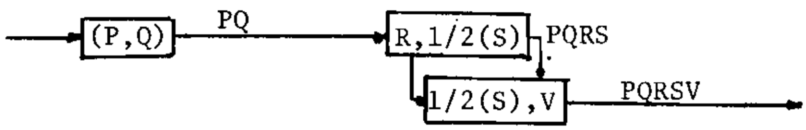
2-cycle paralleling option.

Constructing feasible options:

Option 1



Option 2



Screening: Duplicating V is relatively expensive (approximately \$400). It is not clear which option is the most convenient. Note that for Option 1 sharing of the equipment for V, rather than duplicating the equipment, is a real alternative. The costs of doing this would need to be assessed (In the KWP no such alternatives are explicitely incorporated. This may be an objective for future research). The final decision of which option to choose will not be reached here.

Decision Iteration 5: Assign $\{(P,Q),R,S,V\}$ to Option 1 or Option 2.

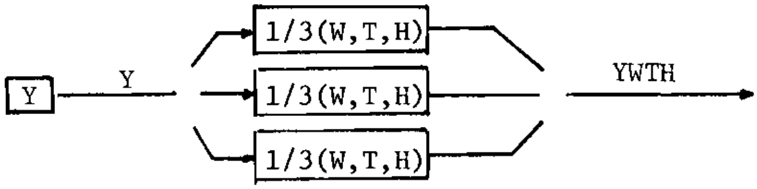
Stations so far assigned: 12 I.T. incurred so far: 35

Iteration 6

A 4-cycle fit is the first acceptable fit obtained. It is $\{Y,(W,T,H)\}$ with I.T. = 120 - 24 - 90 = 6.

A c-3c distribution is obtained: $\{Y\}$ - $\{(W,T,H)\}$.

(W,T,H) is arranged in three parallel stations:



Decision Iteration 6: Assign {Y,(W,T,H)} to the 4-cycle option shown above.

Stations assigned so far: 16 I.T. incurred so far: 41

Iteration 7

The only 1-cycle fit is {X} with I.T. = 5. It is not acceptable.

The only 2-cycle fit is {X, (Z2,Z3)} with I.T. = 60 - 25 - 17 = 18. It is not acceptable.

No other tasks are available. The 1-cycle fit is the best of the two fits.

Decision Iteration 7: Assign task X to a conventional station

Stations assigned so far: 17 I.T. incurred so far: 46

Iteration 8

The only task remaining is (Z2,Z3).

Decision Iteration 8: Assign (Z2,Z3) to a conventional station.

Total stations assigned: 18 Total I.T. incurred: 59

The target of 18 stations have been achieved.

Figure 17 shows the solution obtained.

If at Iteration 3, a 4-cycle paralleling option involving tasks {(K,L),(M,N),(I,J,O),F,G} had been arranged as feasible, a balance of 18 stations would have been obtained. Such balance was actually developed.

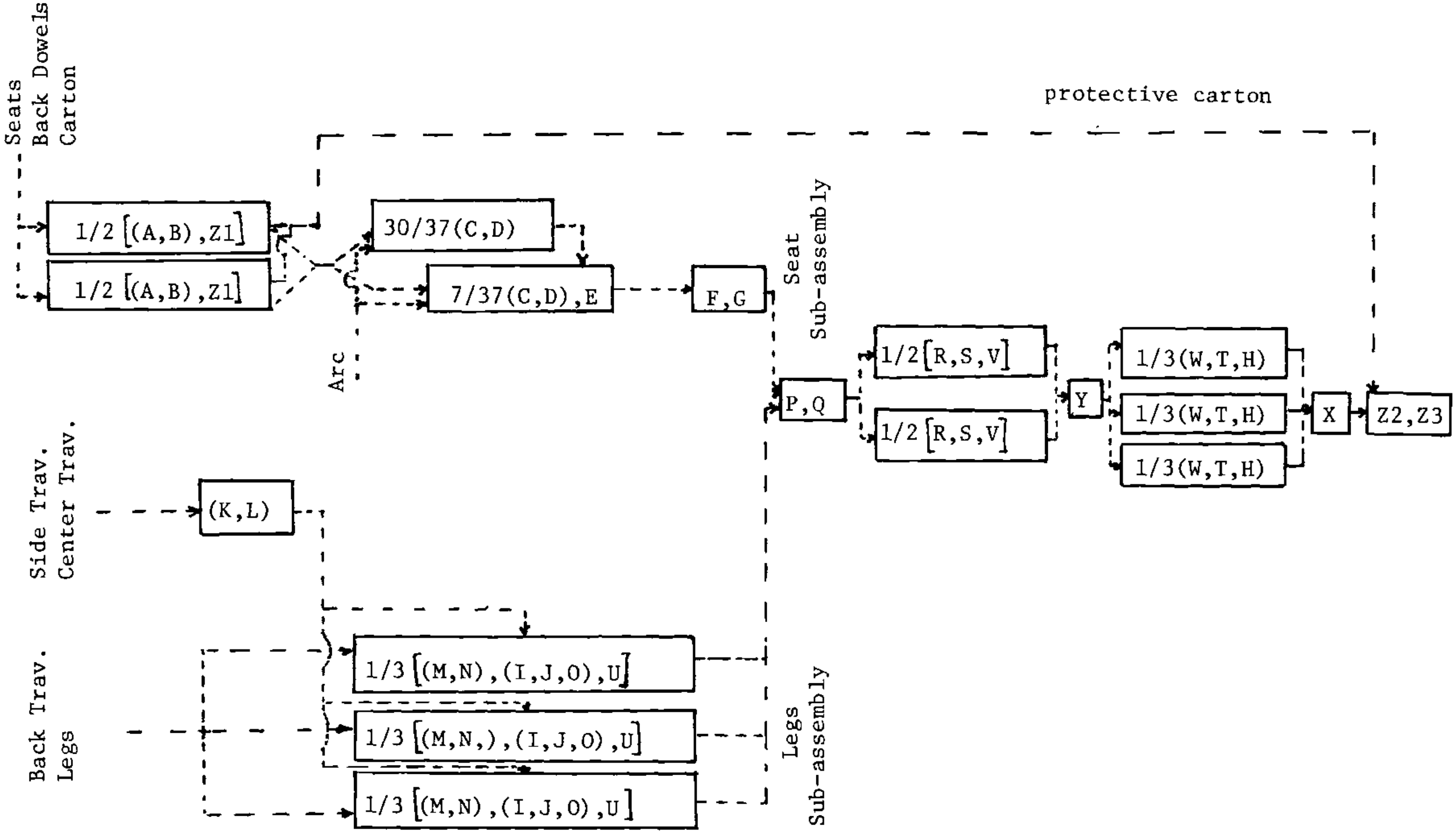


Figure 17. Eighteen Stations for the Windsor Chair.

CHAPTER VII

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary

Line balancing procedures to date are not adequate for many unpaced lines because of the unrealistic restrictions imposed by the procedures. This thesis attempts a more realistic conceptualization of line balancing allowing for duplication of facilities (paralleling).

In Chapter III a terminology and symbology related to paralleling was developed. Also in that Chapter new concepts were introduced and some principles were proposed to limit and guide the search for practical options involving paralleling. In Chapter IV the costs of paralleling were discussed and a heuristic rule, a screening approach, and cost methods were proposed for use in selecting one final option from a group of "simple" feasible arrangements of a set of tasks in a certain number of stations. In Chapter V a heuristic method for line balancing with paralleling was proposed, which is based on the heuristic developed by Kilbridge and Wester to select the tasks available for assignment. The method was presented in a flow diagram with accompanying explanations, and it was applied to an hypothetical precedence diagram. The objective of the new method is to obtain a balance with at most a certain number of stations while tending to minimize costs, and allowing for paralleling. In Chapter VI the method was applied to a real case, the assembly of a chair.

Conclusions

From the application of the KWP to the Windsor chair example, the following conclusions result:

1. A balance consisting of no more than the target maximum number of stations was obtained.
2. Eighteen stations were obtained, twelve of which are non-conventional. Most of the non-conventional stations are necessary anyway, because of the existence of tasks longer than the cycle time.
3. The "simple paralleling options" presented in Chapter III were successfully used to generate logical paralleling alternatives at each iteration of the KWP.
4. In most iterations of the example, the final selection of an option from those considered feasible was made by cost screening rather than more complex cost analyses.
5. Unprogrammed judgmental decisions are sometimes necessary within the context of the KWP when dealing with a real problem. The undesirability of assigning a specific "acceptable" (acceptable according to the idle time criterion) N-cycle fit to N stations represents one such situation.
6. The heuristic column rule used in the KWP to search for N-cycle fits is not very advantageous in cases such as the Windsor chair, where the precedence diagram is relatively inflexible with few tasks at each column and with tasks for two different sub-assemblies situated in the same columns.

7. It would be useful to enlarge the KWP so that it would systematically consider alternatives in which a facility would be shared by two or more workers. Sharing is a real alternative in some cases where the facility is expensive to duplicate.

8. When two or more different parts (or subassemblies) of one final unit are flowing in or out of a station, care should be taken to ensure that the options are well depicted. As explained in Chapter III, the arrows represent units at a given stage of production entering and leaving a station and <u>not</u> different parts of a unit entering or leaving a station.

9. The KWP is not simple to apply. It requires:
    a. Comprehension of several new concepts and principles.
    b. Experience and knowledge necessary to make cost comparisons among options.
    c. Discipline and patience necessary to follow the rather complex step-by-step procedure, and some creativity to identify options.

10. Two very similar problems may vary considerably in the results obtained and effort needed to apply the KWP. This was the case with the Windsor chair example. The same precedence diagram with two changes was balanced for the same cycle time using the KWP. Even though 18 stations were obtained, eight of them were different from those obtained for the precedence diagram shown in Chapter VI. Also, the procedure worked more

smoothly (less effort) for the "two change version" of the precedence diagram.

11. Finally, for a particular problem the KWP could be applied several times, each time for a maximum allowable number of stations. Forward and reverse balancing for each case is also possible.

Recommendations

Some topics recommended for future research are as follows:

1. In this thesis an enlargement of the Kilbridge and Wester heuristic method was proposed for line balancing with paralleling. An objective of future research would be to incorporate the general concepts developed here (e.g., simple paralleling options, N-cycle fit, distribution of an N-cycle fit) in other methods for line balancing with paralleling.
2. In the KWP, it may be useful to consider having a flexible "acceptable fit criterion." At any iteration, if the I.T. being carried from previously assigned stations is less than a certain value (which would be a function of the number of stations assigned), the "acceptable fit criterion" may be relaxed in some way, allowing *in that iteration* for acceptable fits with a percent I.T larger than allowed by the initial criterion. This would reduce the tendency for paralleling when it is not as necessary to parallel to achieve the maximum allowable number of stations.

3. Another area for future research would be the design of computer programs for line balancing with KWP. Interactive systems seem to be most appropriate for this.
4. Enlargement of the KWP with systematic consideration of facility sharing alternatives (see Conclusion 7).

APPENDIX

THE KILBRIDGE AND WESTER METHOD ENLARGED WITH COMSOAL'S HEURISTIC FOR TASKS LONGER THAN THE CYCLE TIME

In order to obtain an initial solution for problems containing tasks longer than the cycle time, the K-W method was enlarged with Comsoal's heuristics.

In this enlarged method the K-W rules and properties are applied at each iteration while searching for 1-cycle fits. When a long task becomes available for assignment and

$$\text{int } \frac{t_{long}}{c} \times c + \underset{\text{at station}}{\text{time remaining}} \geq t \text{ long}$$

then, the long task is considered available for assignment at that station. N-cycle fits, including t_{long}, are considered. N should be the minimum necessary to accommodate t_{long} in the fit. This makes N = $\frac{t_{long}}{c}$, if $\frac{t_{long}}{c}$ is an integer, or N = int $\frac{t_{long}}{c}$ + 1 otherwise.

An initial criterion of acceptable fits is stated ($\approx$ 10% I.T.). Only acceptable 1-cycle fits are sought, except when a long task becomes available.

If the problem is real, ordinal comparisons and min. $\frac{\text{Cost Measure}}{\Sigma t_i}$ would select a final option among the acceptable fits. If no acceptable fit is found, the best fit is selected.

In the hypothetical example of Chapter V with no cost data, the best fit in I.T. was always chosen.

Any assigned fit involving tasks longer than the cycle time is arranged in parallel stations.

For further reference about Comsoal's heuristic see[51].

LITERATURE CITED

1. Moodie, C.L., Young, H.H., "A Heuristic Method of Assembly Line Balancing for Assumptions of Constant or Variable Work Element Times," Journal of Industrial Engineering, 16, 1, 1965, pp 23-29.

2. Freeman, D.R., "Balancing Stochastic Lines," Unpublished Ph.D. Dissertation Stanford University, Stanford, 1967.

3. Kilbridge, M.D., Wester, L., "A Heuristic Method of Assembly Line Balancing," Journal of Industrial Engineering, 12, 4, 1961, pp 292-298.

4. Salveson, M.E., "The Assembly Line Balancing Problem," Transactions of the ASME, August 1955, pp 939-947.

5. Freeman, J.R., Jucker, J.V., "The Line Balancing Problem," Journal of Industrial Engineering, Vol. 18, No. 6, June 1967, page 362.

6. Salveson, M.E., "The Assembly Line Balancing Problem," Transactions of the ASME, August 1955, page 941.

7. Jakcson, J.R., "A Computing Procedure for a Line Balancing Problem," Management Science, 2, 3, 1956, pp 261-271.

8. Held, M., Karp, R.M., Sharesian, R., "Assembly Line Balancing - Dynamic Programming with Precedence Constraints," Operations Research, 11, 3, 1963, pp 442-459.

9. Klein, M., "On Assembly Line Balancing," Operations Research, 11, 2, 1963, pp 274-281.

10. Gutjahr, A.L., Nemhauser, G.L., "An Algorithm for the Line Balancing Problem," Management Science, 11, 1964, pp 308-315.

11. Kilbridge, M.D., Wester, L., "A Heuristic Method of Assembly Line Balancing," Journal of Industrial Engineering, 12, 4, 1961, pp 292-298.

12. Helgeson, W.P., Birnie, D.P., "Assembly LIne Balancing Using the Ranked Positional Weight Technqiue," Journal of Industrial Engineering, 12, 6, 1961, pp 394-398.

13. Arcus, A.L., "An Analysis of a Computer Method of Sequencing Assembly Line Operations," Ph.D. Dissertation University of California, Berkeley, 1963.

14. Hoffman, T.R., "Assembly Line Balancing with a Precedence Matrix," Management Science, Vol. 9, Number 4, July 1963, pp 551-63.

15. Mansoor, E.M., "Assembly LIne Balancing - An Improvement on the Ranked Positional Weight Technique," Journal of Industrial Engineering, Volume 12, Number 4, July-August, 1961, pp 274-281.

16. Moodie, C.L., Young, H.H., "A Heuristic Method of Assembly Line Balancing for Assumptions of Constant or Variable Work Element Times," Journal of Industrial Engineering, Volume 16, Number 1, January-February, 1965, pp 23-29.

17. Sawyer, J.F.H., "Line Balancing," Machinery Publishing Co., 1970, Chapter 2.

18. Mariotti, J., "Four Approaches to Manual Assembly Line Balancing," Industrial Engineering, June 1970, pp 35-40.

19. Wild, R., "Mass Production Management," John Wiley and Sons, 1972, page 49.

20. Salveson, M.E., "The Assembly Line Balancing Problem," Transactions of the ASME, August 1955, page 940.

21. Ibid.

22. Buxey, G.M., "Assembly Line Balancing with Multiple Stations," Management Science, Vol 20, No. 6, February 1974, page 1010.

23. Lehman, M., "What's Going on in Product Assembly," Industrial Engineering, April 1969, pp 41-45.

24. Buxey, G.M., Assembly Line Balancing with Multiple Stations," Management Science, Vol. 20, No. 6, February 1974, page 1010.

25. Salveson, M.E., "The Assembly Line Balancing Problem," Transactions of the ASME, August 1955, pp 939-947.

26. Kilbridge, M.D., Wester, L., "A Heuristic Method of Assembly Line Balancing," Journal of Industrial Engineering, 12, 4, 1961, pp 292-298.

27. Gutjahr, A.L., Nemhauser, G.L., "An Algorithm for the Line Balancing Problem," Management Science, 11, 1964, pp 308-315.

28. Arcus, A.L., "An Analysis of a Computer Method of Sequencing Assembly Line Operations," Ph.D. Dissertation University of California, Berkeley, 1963.

29. Ibid., page 159.

30. Freeman, J.R., Jucker, J.V., "The Line Balancing Problem," Journal of Industrial Engienering, Vol. 18, No. 6, June 1967, pp 361-364.

31. Freeman, D.R., "Balancing Stochastic Lines," Unpublished Ph.D. Dissertation Stanford University, Stanford, 1967.

32. Heskiaoff, H., "An Heuristic Method of Balancing Assembly Lines," Western Electric Engineer, October 1968, pp 9-17.

33. Helgeson, W.P., Birnie, D.P., "Assembly Line Balancing Using the Ranked Positional Weight Technique," Journal of Industrial Engineering, 12, 6, 1961 pp 394-398.

34. Heskiaoff, H., "An Heuristic Method of Balancing Assembly Lines," Western Electric Engineer, October 1968, p 15.

35. Mariotti, J., "Four Approaches to Manual Assembly Line Balancing," Industrial Engineering, June 1970, pp 35-40.

36. Buxey, G.M., Slack, N.D., Wild, R., "Production Flow Line Systems Design - A Review," AIIE Transactions, Vol. 5, No. 1, March 1973, pp 37-48.

37. Ibid., page 39.

38. Ibid., page 38.

39. Buxey, G.M., "Assembly Line Balancing with Multiple Stations," Management Science, Vol. 20, No. 6, February 1974, page 1010.

40. Helgeson, W.P., Birnie, D.P., "Assembly Line Balancing Using the Ranked Positional Weight Technique," Journal of Industrial Engineering, 12, 6, 1961, pp 394-398.

41. Lehman, M., "What's Going on in Product Assembly," Industrial Engineering, April 1969, pp 41-45.

42. Pinto, P.A., Dannenbring, D.G., Khumawala, B.K., "A Branch and Bound Algorithm for Assembly Line Balancing with Paralleling," Int. J. Prod. Res., Vol. 13, No. 2, 1975, pp 183-196.

43. Pinto, P.A., "Assembly Line Balancing with Paralleling," Ph.D. Dissertation, The University of North Carolina at Chapel Hill, Chapel Hill, 1975.

44. Wild, R., "On the Selection of Mass Production Systems," Int. J. Prod. Res., Vol. 13, No. 5, pp 443-461, 1975.

45. Wild, R., Slack, N.D., "The Operating Characteristics of Single and Double Non-Mechanical Flow Line Systems," Int. J. Prod. Res., Vol. 11, No. 2, 1973, pp 139-145.

46. Kilbridge, M.D., Wester, L., "A Heuristic Method of Assembly Line Balancing," Journal of Industrial Engineering, 12, 4, 1961, pp 292-298.

47. Jackson, J.R., "A Computing Procedure to a Line Balancing Problem," Management Science, 2, 3, 1956, pp 262-265.

48. Kilbridge, M.D., Wester, L., "A Heuristic Method of Assembly Line Balancing," Journal of Industrial Engineering, 12, 4, 1961, pp 294-291.

49. Arcus, A.L., "An Analysis of a Computer Method of Sequencing Assembly Line Operations," Ph.D. Dissertation, University of California, Berkeley, 1963, page 186.

50. Ibid., pp 99-102.

51. Ibid.